Strategies for Developing Mathematics Skills in Students Who Use Braille

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August, 1997

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Dr. Larry Chang, of the University of California, was also very generous in contributing his *Speakeasy*, a system he developed for the consistent reading of mathematical symbols, especially in the areas of algebra, geometry, trigonometry and calculus. His *Speakeasy* is presented in its entirety in Appendix B.

Mr. Mario Cortesi, of the Chicago Public Schools, the major author of the tutorial entitled Computer-assisted Instruction for Learning the Code of Braille Mathematics, has contributed numerous creative ideas for involving students in higher mathematical experiences. He has also contributed a set of quick reference sheets which are included in the tutorial. The reference sheets are contained in Appendix C.

Innovative ideas and pertinent issues and questions were contributed by numerous teachers who participated in several focus groups and/or who responded to a national survey regarding several of the topics addressed in this manual. These individuals are named in Appendix F.

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INTRODUCTION

Barriers to Achievement in Mathematics

With the notable exception of the article reprinted in Appendix A (included for its interesting historical perspective), a thorough review of the literature reveals that achievement in mathematics among blind and severely visually disabled persons is, and always has been, extraordinarily low. There are several reasons for this unfortunate situation. The first and perhaps most important reason is that mathematics is very visual in nature. Visual reference is the basis for much of the language of mathematics, with the description of such things as direction, quantity, and shape as fundamental elements. The development of spatial and directional concepts, as well as understanding of the concepts of object permanence and conservation of mass and volume are often delayed in children who are congenitally blind; and the blind student must piece together information which is perceived as a whole, in its entirety, by the sighted student.

For a person who has no sight or very little useful sight, the study of mathematics is difficult. It requires considerably more effort on the part of the blind student than does the study of fields which are more verbal in nature. Generally, in order to achieve at reasonably high levels in mathematics, blind students must possess greater aptitude for the subject than their sighted counterparts. Since mathematics is difficult for blind persons to learn, students are unlikely to demand more emphasis on the subject. Younger students, of course, have no knowledge of the fact that they are not receiving sufficient training.

A second major reason for poor achievement in mathematics among blind students is that teachers of visually impaired students and rehabilitation specialists often lack skill and knowledge in the area of mathematics instruction. Many have had inadequate preparation in the Nemeth Code. While it is logical for personnel preparation programs to concentrate on the literary braille code, instruction in the Nemeth Code is often relegated to a subordinate position in the array of skills and knowledge. Consequently, many teachers lack confidence in their ability to teach the reading and writing of braille symbols in mathematics; this becomes a neglected area of instruction.

A third reason for inadequate mathematics instruction is that only a small minority of individuals who join the field of special education and rehabilitation have technical backgrounds in which mathematics was a major portion of their study. Many teachers and rehabilitation specialists have concerns about their personal level of mastery of the field. There is a natural tendency in the art and practice of teaching to place emphasis on areas which are of interest to the instructor,

areas in which the instructor has expertise. Mathematics, therefore, usually is not the focus of instruction.

Impact of Restricted Development of Mathematical Skills

Insufficient preparation in mathematics has a profound effect upon blind persons. The disadvantages in daily life are numerous, including difficulties and unnecessary limitations in normal tasks of daily life as well as educational and occupational opportunities. While it is commonly agreed that the level of mathematics achievement required for daily living is not high, many blind persons, in reality, are functionally illiterate in mathematics. For example, it is not uncommon to find blind or severely visually disabled persons who cannot determine the correct change which should be received when making a purchase, calculate the amount of interest one might pay on a loan, or add one-half cup and three-fourths cup to accurately measure ingredients in a recipe. Given "real life" tasks, many cannot choose the basic mathematics operation or combination of operations to solve a common problem.

Inability to achieve in mathematics has a deleterious effect upon educational opportunities afforded to blind persons. Most entrance examinations contain a quantitative subtest. Scores from that subtest are included in the calculation of the overall score, representing the examinee's performance and aptitude. If fundamental mathematics skills are poorly developed, then the general aptitude score will be depressed, impeding entrance into higher education or professional programs. In a modern technological society, well-honed quantitative skills are very important. Because most blind persons have poorly developed mathematics skills, they tend to avoid technical areas of employment. In fact, many technical areas are closed to them because they are unable to demonstrate the fundamental level of mathematics competence necessary for entrance into such fields.

Factors Critical to Success

Professional development

One solution to this problem is better trained teachers and rehabilitation specialists. If these individuals have confidence and the technical expertise to provide the necessary instruction, many more blind persons in the future will experience success and competence in the essential and critical area of mathematics.

Given the paucity of resources designed to help teachers remain current in methods and materials for instruction in mathematics for blind students, the following are sources of information which may be helpful. Catalogs of non-profit as well as for-profit organizations which produce materials and devices for use by blind students

are a very useful source of information. The American Printing House for the Blind (APH), located in Louisville, Kentucky, is one of the nation's leading producers of instructional materials and devices for use by blind persons. Exceptional Teaching Aids (ETA) of Castro Valley, California is an excellent example of a for-profit corporation which provides an assortment of materials and devices.

Other excellent sources of materials are catalogs of teaching materials meant for use by sighted students. With the emphasis upon tangible devices for teaching mathematics, many potentially useful tools and materials which will require little or no adaptation can be found in these catalogs. Other examples of companies or organizations can be found in the Resources section of this manual.

Consultation and brainstorming with teachers who specialize in mathematics instruction in the regular classroom can generate ideas for effective strategies and appropriate methods for instruction of blind students. Consultation with experienced teachers of blind youngsters who have established a record of achievement in their work is also an excellent source of valuable information and advice. Many of these individuals can be found in residential schools for blind youngsters. These institutions often employ individuals who have specialized in mathematics and who also have been trained in the special methods for instructing blind youngsters. Large, urban school districts are likely to have employees who are certified teachers of visually impaired students with extensive experience teaching older blind students who are required to complete advanced mathematics courses.

Colleges and universities usually provide services for students with disabilities; some employ persons who specialize in working with blind students on their campuses. Many of these specialists have discovered solutions to the difficulties faced by blind persons who must complete college level mathematics courses. Therefore, general education teachers who are specialists in mathematics, teachers of visually impaired students who teach mathematics in residential schools or large urban school districts, and college disability services personnel, with their well-developed experiential backgrounds, can be a source of valuable assistance in solving some of the problems which may be faced by other teachers in their work with blind students.

A rapidly expanding source of information is the Internet. There are numerous websites, listed in the Resources section of this manual, which currently address mathematics and science issues specifically for visually impaired students. There are hundreds of sites which address mathematics in general; a few are listed in the Resources section, but a general exploration of this topic would yield countless additional sites and information or suggestions on particular areas of mathematics.

In order to provide effective instruction in mathematics, teachers need a broad base of skills and information in the concepts as well as potential adaptations and

modifications in measurement, graphics, geometric designs, charts, figures, use of the calculator, compass, and protractor. Inservice workshops conducted by individuals who are knowledgeable in these areas represent another potentially valuable source of information; unfortunately, these are rare. The scarcity of such workshops is evidence of the inadequate number of individuals in the nation who are competent in providing instruction in mathematics for blind students.

A teacher who needs to review mathematics concepts can easily do so through the use of one of the many self-teaching guides available in bookstores. An example of a quick review of essential concepts is *All the Mathematics You'll Ever Need*, by Steve Slavin (1989). There are many books, widely available, which provide programmed instruction and review in specific subject areas, from algebra and geometry to statistics and calculus.

A personal perspective

On March 4, 1996, the following response to an Internet survey on mathematics instruction for blind students was sent by Abraham Nemeth, addressing a personal perspective of factors which were critical to his ultimate achievement and success as a mathematician:

> I did not require any special methods of instruction. I attended regular classes where I took notes in braille supplemented by tape recordings of the lectures. I developed the Nemeth Code when I found that the other available codes in existence were inadequate as a tool for doing mathematics.
>
> I used the braillewriter as the exclusive device by which I performed mathematical calculations and manipulated mathematics expressions. On a braillewriter, the dots appear on the top side of the paper where they can be immediately read. This allows for the rapid alternation between reading and writing which is required when interacting with mathematical expressions, and is the closest thing to the use of a pencil and paper used by the sighted.
>
> To operate effectively in this manner, one must be a skilled and accurate braille user. I am congenitally and totally blind and was fortunate to have been taught braille at an early age. There were no areas in mathematics that caused me any particular difficulty. I often wished that my instructor would verbalize what he was writing on the blackboard. I tried contacting my professors to ask them to do that; sometimes I was successful and sometimes not. I believe that I could not have reached my potential in mathematics without the Nemeth Code. With it, I am able to read and

write mathematics, as well as other sciences, at all levels, limited only by my talent and my ambition.

Alleviation of mathematics anxiety

A student who has experienced frequent failure in mathematics is unlikely to be motivated to improve his or her performance. It is probable that he or she attributes the failure to bad luck, or difficulty of the subject. The student is likely to believe that increased effort and persistence will not make any difference in the outcome, that he or she has no control over success or failure in mathematics; and to develop a stance of helplessness and passivity (Corral, 1997).

To help change this perception, instruction in actual learning strategies specific to each type of mathematical operation or concept, paired with instruction in positive “self-talk” can be effective. The student will begin to expect success instead of failure, and to see the connection between effort and success.

Learning strategies

Depending upon the specific circumstances, one or more of the following learning strategies will help a student to master mathematics skills, and to apply those skills to solve real-life problems: use of manipulatives; estimation; simplification of problems through a separation into subunits; elimination of extraneous information; or verbalizing the problem using if-then logic.

It may be helpful for students to develop an organizer, and also a checklist of questions to aid in systematic problem solving as well as the self-checking process. The student can refer to his or her list while solving a problem, until the steps are covered without this reference. An example of an organizer is RAPS (Meltzer, 1996):

- R=READ & RAP (read problem and repeat in your own words)
- A=ART (draw a diagram or use objects to show the problem)
- P=PLAN & PREDICT (think of a plan for solving the problem and predict or estimate the answer)
- S=SYLVIA (check answer with calculator, which in this case is named Sylvia)

A checklist could include these sample questions covering typical errors:

1. Are the correct numbers written in the problem?
2. Are the numbers lined up correctly?
3. Are the signs of operation correct?
4. What is my estimated answer?

5. What is the actual answer? Is it reasonable? Is it close to my estimate?

Positive attitude

Following are some suggestions for teaching a student to think positively about mathematics:

- Model each learning or problem solving strategy (examples, as listed above, or specific to a particular curriculum or student), with a written reference, stressing the reasons it is valuable:
    1. Model the steps by working through a problem verbally, explaining the strategies used at each step.
    2. Work another problem along with the student, continuing to verbalize strategies and their value.
    3. Have the student work through a problem while stating the strategy steps; provide immediate corrective feedback.
    4. Have the student work through a problem without stating the strategy steps; provide immediate feedback.
    5. Next, the student will solve several problems independently.
    6. Finally, the student will state the strategy steps from memory, and demonstrate how they are used.
- Model the use of positive statements while working on a mathematics problem (Corral, 1997); for example:
    1. I can probably solve this problem because I have been successful solving problems which are very much like this one.
    2. If this problem seems difficult, that means I need to try harder; then I will probably be successful.
    3. If I work carefully, I will probably be successful.
    4. If I make a mistake, I will be able to find it and correct it.

Curriculum and Standards

In 1989, the National Council of Teachers of Mathematics produced *Curriculum and Evaluation Standards*, a policy document developed on the basis of 20 years of research and curriculum development. Addressing the consensus opinion of mathematics educators, the document serves as a structure for reform in mathematics education, recommending higher standards for student performance and a more challenging "discovery-oriented" curriculum, with an emphasis on understanding of concepts and solving longer, more complex problems (Woodward & Baxter, 1997). This is achieved through a reduction of time and effort on practice in computation, and

more emphasis on concepts, exploration, problem solving without the use of key words (e.g., “took away”, “bought more”) to imply operation, and defense or explanation of problem solving methods.

Reformed methods and standards in mathematics education, emphasizing meaningful problem solving and multiple, effective strategies for solving problems rather than memorizing facts, makes speed a critical factor in student success. Students need to be able to recall mathematics facts instantly, and to decide which tools would be appropriate to solve a problem. There has to be a balance between rote learning of facts and learning of problem solving strategies through logical reasoning; that is, a balance between mathematics skills and mathematics strategies.

This current standard may pose additional problems for special needs students who are included in the regular classroom, since it proposes teaching concepts more quickly and in greater depth, with less repetition, and using real-life problems from other curricular areas which require multiple-step solutions. It is necessary to devote considerable time to developing and practicing skills of pattern identification, number sense, estimation, and developing multiple solutions for problems, while using an array of manipulatives and calculation tools. Rote skills need to be combined with challenging problems which apply the skills in order to prepare adequately for instruction in higher mathematics.

For blind children, pre-teaching of mathematics concepts is very important, especially when students are learning basic concepts, language, and Nemeth Code. Particular emphasis should be placed on teaching students to be self-advocates, articulating what a classroom teacher needs to do in order for the student to participate in learning. Students should also be taught about spoken mathematics, and how to teach a peer or reader to speak the language. They may develop their own ideas, or have access to a system previously developed (for example, refer to *Handbook for Spoken Mathematics*, Appendix B).

Following are some generalizations which the teacher of visually impaired students should bring to the attention of the classroom mathematics teacher, to facilitate meaningful class involvement and participation by the blind student:

- During a lecture, words such as “this”, “that”, and “there” will be meaningless to a blind student and should be avoided.
- Description of problems or techniques should be worded carefully to avoid ambiguity; a copy of the *Handbook for Spoken Mathematics* (Chang, 1983), or a copy of relevant pages from this resource, may be helpful.
- When writing on the chalkboard, verbalize what is written; be sure to describe labeling of a diagram drawn on the board, and spell new words while they are being written.

- Provide transparencies and notes from the chalkboard for the teacher of visually impaired students to transcribe into braille for the blind student to use at his or her desk. Ideally, these can be sent home in print and braille prior to a given lesson, so they can be previewed, then reviewed, by the student and his or her parent.
- When explaining concepts through the use of everyday objects, be careful to choose objects which a blind student is able to access and understand. Things which can be explored tactually and in entirety, such as hardware, toys, kitchen utensils, etc., will have meaning, whereas large and/or remote things such as airplanes, elephants, clouds, etc., will be difficult to understand.
- It is often helpful for a blind student to have a print copy of textbooks and handouts to be used by a reader at home.
- The blind student sometimes needs extra desk space, and a storage area for braille materials.
- If students are completing problems on the chalkboard, another student can write the work as the blind student explains it aloud.
- Tests can be administered orally. Occasionally, an oral test may be given to the entire class; usually, it will be given to the blind student individually, outside of the regular classroom. Tests can also be recorded; the blind student can use headphones and remain in the classroom to take the test.
- Mathematics assignments should be checked daily by someone who can actually read the braille. Braille answer sheets should be sent home so parents can check the student's homework. Assignments and answer keys need to be forwarded to the teacher of visually impaired students in advance.
- Teach classroom teachers how to make simple adaptations, such as using a tracing wheel or Wikki Stix for graphs which are immediately available to the student.
- Provide the classroom teacher with Nemeth Code "cheat sheets"; he or she may be able to spot check some of the student's work.

Assessment

Subject-centered mathematics assessments will help pinpoint a student's current level of functioning, in addition to providing information about specific areas in which the student needs additional instruction (Meltzer et al, 1996). There are many published tests currently in use, such as the *Brigance Diagnostic Comprehensive Inventory of Basic Skills* (K-9), the *Brigance Diagnostic Inventory of Essential Skills* (4-12), *Key Mathematics* (K-6); general achievement tests such as the *California Achievement Test*, the *Iowa Tests of Basic Skills*, and the *Stanford Achievement Test*,

which have mathematics sub-sections; and tests which are incorporated into mathematics textbooks and instructional materials.

Any of these can be modified, using a combination of braille and oral administration as appropriate, to assess the general level of performance in mathematics. Since some adaptations involve the use of graphic displays, it is important that teachers work with students to move from real objects to models and two dimensional representations so that they are not penalized by such adaptations. It is also important to review tests before their use to insure that examples and graphics are meaningful to the blind student, and to determine the need for manipulatives. An additional note of caution: standardizations do not apply to braille or oral forms of tests; any comparisons to the norms provided should be interpreted with that in mind. Useful information can, however, be obtained by analyzing strengths and areas of difficulty related to concepts and skills addressed in a particular assessment instrument.

A diagnostic teaching approach also provides excellent opportunities to assess students' understanding of certain concepts and their ability to apply the skills they are learning on an ongoing basis. A combination of these assessment approaches can result in specific and functional data. Attention can then be directed to appropriate programming.

It is important that teachers not be misled by their student's verbal fluency or auditory memory, because this may lead to the false assumption that they understand concepts which they actually may not. Having students use their skills to solve problems from everyday situations, explaining the steps they take and the rationale for those steps, will provide evidence of their understanding or lack of it.

Following an analysis of specific areas in mathematics which may require further development, it is important to observe students to determine their mathematics learning style. In order to be successful in learning mathematics, a student must develop the ability to retain (memorize) facts; recognize patterns; recognize relationships (part to whole, sequential and spatial concepts); categorize and classify; organize mathematical information; "sense" numbers to recognize when an answer is reasonable or possible; think through a problem, plan a solution, and predict an outcome; differentiate essential information from that which is superfluous and irrelevant; and use mental flexibility to identify multiple ways to solve a problem.

In addition to the range of mathematics skills that require evaluation, the student's knowledge and use of the Nemeth Code, as well as the use of appropriate calculation tools, must be assessed. This can be done in a number of ways. Checklists such as the *Minnesota Braille Skills Assessment* (McNear, 1995), the *Braille Recognition Level Tests for Mathematics* (Czerwinski, 1982), or those designed by teachers to address specific skills can provide information to document progress or pinpoint areas for instruction. However, ongoing activities such as those listed below can also be helpful:

- Have students orally read problems in Nemeth Code to identify specific symbols which may require attention.
- Use word problems for which the student writes the appropriate number sentence.
- When grading arithmetic classwork or homework, give separate grades for the mathematical calculations and for the use of the Nemeth Code.
- Provide opportunities for several blind students working on the same skills to have "mathematics bowls" for motivation and practice.
- Develop a portfolio of samples of the student's work using Nemeth Code, and/or the braillewriter.
- Have the student talk through the working of problems on the abacus, or the spacing of problems on the braillewriter.
- Use game formats such as card games or board games in which the student a) draws a card from a deck, b) reads the Nemeth Code aloud, and c) works the problem on the braillewriter or the abacus. The student can earn points for correct reading of Nemeth Code, and for the correct use of the calculation tools being evaluated.

Difficulty in mathematics may stem from seemingly unrelated learning problems, including reading skills, language and communication skills (receptive as well as expressive), and attention problems. These difficulties may also create mathematics anxiety which, in turn, can impede learning. If a student is not progressing at a suitable rate, it would be wise to assess these areas. This may be accomplished by thinking through the following questions, as they pertain to a particular student: Does the student . . .

- analyze patterns?
- recognize part-whole relationships?
- identify relevant information; ignore irrelevant information?
- understand mathematics-related vocabulary?
- need additional time to process information before responding?
- understand verbal directions?
- read problems independently?
- confuse the meaning of the written word?
- understand the relationship between words, graphs or line drawings, and objects?

TEACHING MATHEMATICAL CONCEPTS

Number Sense

It is believed by neuropsychologists that humans are born with "number sense", or an innate ability to perceive, process, and manipulate numbers. It is an intuitive ability to attach meaning to numbers and number relationships, to understand the magnitude of numbers as well as the relativity of measurement of numbers, and to use logical reasoning for estimation.

While some ability to understand numbers may be intuitive, this ability is also one which uses many visual referents. The sighted child is able to compare groups of objects and immediately analyze differences and likenesses in amount, size, and other characteristics. The blind child must explore the same groups in parts, and often only after their attention is directed to the task, before being able to draw conclusions about similarities and differences. There are many incidental opportunities for the young sighted child to use this number sense in their daily life; the blind child needs to be directed to, and guided through, these opportunities for exploring, comparing, ordering and problem-solving in the real world to allow for a natural development of number sense. Such opportunities will also cultivate a positive attitude toward mathematics and facilitate the child's achievement and confidence. Following are a few suggestions:

- Encourage children to explore groups of objects which can be perceived with one or two hands (e.g., coins, candy, legos, beads, buttons, pretzels, Cheerios) to compare the relative size of groups of things.
- Provide extensive opportunities to match number of objects to number of fingers.
- Talk about numbers: how many, how many more or less, how many more are needed.
- Assign number names to groups of objects which are dissimilar in size or shape, for experience with the concept of quantity and comparison of quantity.

Basic Concepts

For students to understand and work with formal mathematical concepts successfully, they must understand the concepts of classification, conservation, seriation, ordering and one-to-one correspondence. Students must first work with and understand these concepts on the basis of quality (e.g., attributes such as shape, size, weight) before moving on to their application to general quantity (e.g., attributes such as many, few, none) and then on to number (e.g., attributes such as "fiveness", 100=10X10, 4+1=1+4) (Moore, 1973; Stephens, 1973; Ostad, 1984).

In order for students to develop their innate number sense, and a working knowledge of the above concepts, they must have a great variety of interactions with their environment, exploring and manipulating, comparing, arranging and rearranging real objects and sets of objects. Many of these types of interactions and experiences occur incidentally for sighted children, but the blind child is at great risk for missing valuable and relevant incidental information (Warren, 1984; Stephens, 1972). Therefore, it is critical that teachers and parents provide both structured and informal opportunities to handle and explore, note likenesses and differences, match, group and classify, order, and experience other relationships with real objects to prepare them for understanding the same relationships with numbers.

One of the earliest concepts to be developed is that of classification. Classification involves discrimination, matching, and grouping or categorizing according to attributes and attribute values. A sampling of these attributes and attribute values at the quality level follows:

- Shape (square, circle, triangle, rectangle)
- Size (large, small, big, little)
- Weight (heavy, light)
- Length (short, long)
- Width (wide, narrow, thick, thin)
- Height (tall, short)

At the quantity level, these attributes would involve general number concepts (e.g., many, few, more, less, none), and later, more specific number values (e.g., sets of 2, sets of 10, sets of values greater than 2).

The development of classification concepts involves several sequential stages:

a. discriminating between same and different (note: if a child has difficulty with the dichotomy of same/different, the dichotomy of same/not same may be more effective to begin with); attention should be called to the critical features of objects and their attributes;
b. matching, grouping and categorizing according to specific criteria; and
c. classifying according to a variety of dimensions.

To promote the development of classification concepts, the teachers can:

- Begin working on simple discrimination and matching with objects that are familiar to the child and that occur naturally in his or her world (e.g., shoes, toothbrush, squeeze toys, blocks, etc.), then move on to noting and analyzing specific attributes (e.g., shape, size); later, those specific attributes can

be applied to naturally occuring objects in the environment (e.g., circle shape of a plate).

- Provide numerous opportunities for the child to handle and explore objects, note their critical features or attributes of shape, size, position in space, length, etc.
- Provide many opportunities for the child to match objects, and build groupings or sets of objects on the basis of specific attributes.
- Follow a logical or Piagetian sequence with regard to matching, grouping or categorizing, and later classifying: start with a single criteria or attribute by which to discriminate or group (e.g., shape/circle), change to a different criteria (e.g., small/large), progress to two attributes simultaneously (e.g., small circle), add additional attributes (e.g., small thin circle), and finally discriminate according to attributes NOT present (e.g., item that is not round, not small).

Another basic concept that children must understand is that of seriation, or ordering objects, then quantities, and eventually numbers, according to specific given criteria. As with the concept of classification, the child must begin working in this area with real objects on the basis of quality (e.g., ordering family members' shoes or belts according to attributes such as length). Only then will the child be able to apply the concept to quantity (e.g., ordering jars of coins or chains of keys—one having many, one having several, one having few and one having one or none), and later to number (e.g., ordering the numerals 2, 10, 3, 5). The concepts of classification and seriation can be taught in conjunction with each other very effectively (Ostad, 1984). For example, after the child can match and sort according to size, he or she can work on ordering from largest to smallest.

In addition to the understanding of the concepts of classification and seriation, the child must develop an understanding of conservation—knowing that a given amount remains the same though its appearance may change. Also, as with classification and seriation, the concept of conservation must be developed first with real objects (e.g., a bowl of cake mix is the same amount as when it is divided into 12 cup cakes). This must be understood before a child can be expected to understand the "partners" that make up numbers (10=5+5, 10=7+3, 10=6+4), units of measurement and money (a nickel is the same amount as five pennies), fractions (one whole is the same amount as two halves or four quarters) or the associative principle (7x3 equals the same as 3x7).

In addition to the concepts of classification, seriation, and conservation, children need to understand basic spatial and positional concepts. For example, the concepts of top, bottom, around, middle, center, corner, line, straight, curved, next to, beside, are very relevant to basic mathematical understanding. Later, concepts such as diagonal, parallel, perpendicular, intersecting, angles, and rotating will be relevant.

Positional ordering concepts are also critical for sorting, for seriation, and for working with sets; these include concepts such as first, second, third, next, last, before, and after. However, these concepts require basic counting ability.

When teaching any of these basic concepts, it is important to start with real three dimensional objects, progressing to two dimensional shapes or diagrams and finally to more symbolic representations. It is also advantageous to have students develop the ability to express their discriminations in complete sentences (e.g., "These are the same because they are both square," or "This is the longest belt.") because doing so helps them to focus their attention on the concept rather than simply naming a descriptor (Ostad, 1984).

Activities for teaching basic concepts

- Involve children in daily living activities around the home or classroom. For example, helping to put silverware away in a divided tray with a sample in each section provides practice in matching, sorting and categorizing; helping to sort different sizes of towels or different items of clothing provides additional practice with these concepts.
- Give children numerous opportunities to use everyday items for matching and categorizing: eating utensils, grooming tools, foods, and toys for function; shoes and shoelaces for matching by size or length.
- To work on seriation, have children arrange boots belonging to family or class members from smallest to largest size; boots could also be arranged by height.
- The same type of activity could be carried out with other personal items such as belts of different lengths, books of different thicknesses, milk cartons of different sizes, or later with Unifix towers or Cuisenaire blocks. Students should not only identify the "extremes" of a series (e.g., longest or shortest), but also the "next shorter".
- Having family members or class members line up according to height can also help to facilitate understanding of seriation.
- Provide chances for children to work with the concept of conservation: give them a ball of clay and let them divide it into smaller amounts as they wish, and then combine the smaller shapes to demonstrate the constancy of amount.
- Using the sorting tray from Science Activities for the Visually Impaired (SAVI) or from APH, place a variety of small items (buttons, paper clips, keys) in the larger section; to categorize, place one of each type of item in each of the smaller sections of the tray and have the child match and sort the remaining items; to classify, have the child form his or her own groups

without providing a model. This activity could also be done using attribute blocks.

- Have children fold stiff fabric and paper to make different shapes. Squares can be folded to make triangles or smaller squares. Later, origami can be used to facilitate understanding of geometry.
- Children can explore shapes and size by building with Legos and Unifix blocks; they can also work with conservation by making a variety of different groupings from a given number of blocks.
- Have children copy simple shapes on geoboards; later they can make their own shapes based on names or clues such as "four corners", etc.
- Provide children with opportunities to explore and compare the three-dimensional shapes from Essential Geometric Forms (APH).
- Have children walk, hop, run, jump through an obstacle course made from large shapes on frames, available from several children's catalogs, or arranged from items in the natural environment (e.g., jump 3 times in the circle, hop through the square, step in and out of the triangle).
- Use shapes, sizes, orders, patterns, planes, and eventually numbers in the real life environment (classroom, home) to teach concepts (e.g., compare the size of books to each other and to the size of tables, use corners of rooms to demonstrate angles, etc.).
- To practice positional ordering, have a student line up the rest of the children in a group, and then identify each as first, second, third, . . . last. Also have the student identify which child is before or after a particular individual, which one is next, etc. Children can also do the same activity by arranging toy cars or other manipulatives.
- Make a mathematical "pattern block" to enable students to build shapes and patterns with manipulatives that stay in place. To make the pattern block, drill ten or twelve evenly spaced holes into a long block (22" x 3") such as the ones found in kindergarten block centers. Hammer thin wooden dowels or glue pieces of thick stranded wire into the holes, leaving about 2" protruding up out of the block. Assemble a collection of small objects that slide easily over the dowels or wires (e.g., beads of various sizes/shapes, washers, straws, plastic Unifix cubes, large paperclips, uncooked pasta, small pretzels). Students slide objects over the dowels in a left to right sequence to make a pattern (cube, cube, pretzel, cube, cube, pretzel, etc). The teacher can also start a pattern and have the student finish it. This device can also be used to teach ordinal number positions such as first, second, next, last.
- Use magnet boards or felt boards for children to match shapes, size, position, order, and patterns; later, children can match numbers or form

simple number statements to accompany the arrangement of manipulatives.

One-to-One Correspondence and Counting Skills

In addition to the basic concepts discussed above, understanding the one-to-one correspondence of object to object is also necessary before the child can carry out meaningful counting and higher calculations.

Children can find many opportunities in their daily life to experience one-to-one correspondence. They can place one sock inside one shoe or one shoe on one foot; they can get one napkin or snack for each member of the family or class; they can place one lid on each of several containers; they can place pieces in one-piece puzzles.

Once children understand these relationships, they can correspond one number with one object and then count with understanding. "Rote memorization of a set of numbers is meaningless" (Moore, 1973, p. 67) and counting is a skill which should not be stressed until the child has shown understanding of basic classification, conservation, seriation and set comparison at both the quality level (attributes of objects) and the quantity level (general amounts in groups or sets).

When students are ready to develop the skill of counting, they can benefit from learning several counting strategies to increase their accuracy and efficiency. Students sometimes develop one or more such strategies on their own, but it is to their benefit to provide training in this area. As with any concepts or skills, it is important to start working with real objects and manipulatives and to continue providing these as learning aids.

Objects to be counted are often found in one of several types of arrays: linear, circular, rectangular, or random. The following steps can be helpful for young children in identifying the counting situation, organizing it, and keeping track of their progress as they count the items in the array.

1. *Scanning*—The child moves his hands across the top of each item in the array to be counted, in order to obtain information about the objects and the general field over which they are spread. The child could also pick up and examine items and replace them in the tray.
2. *Organizing*—If items are randomly displayed, the child can move all items to one side in preparation for counting. If items are already arranged in a linear fashion, the child can locate the first item in the series and scan to confirm the arrangement.
3. *Partitioning*—The child can count individual items and move counted items to a separate area on the tray. The child could also pick up items one at a time, give them a name, and replace them apart from those yet to be

counted. The child could also individually touch each item to be counted with one hand, giving each a numeral name, while the other hand keeps track of the next item to be counted.

When teaching counting skills, these suggestions might help:

- Pair word problems with calculations at the earliest levels, even if it involves only an easy oral "story" problem to go with sets being counted.
- As soon as possible, tie the use of manipulatives and oral counting and number statements to the representation of these numbers on paper with the brailler and on the abacus. Use manipulatives alongside these tools during transition to the brailler and the abacus.
- Provide the student with notes on basic number concepts; these can be kept simple with examples to illustrate. A small flip chart such as those available in teacher stores could be labeled in braille.
- Modify a braille meter stick by covering it with clear braillon; re-label it with 0 in the middle, 0-50 going to the right, and negative numbers moving to the left. A similar modification could be made to a raised line ruler.

Activities for teaching counting

- Have the child compare/match/sort groups of objects into sets; then have him or her identify the number of items in each set, expressing them by name and by some pattern (e.g., clapping or ringing a bell the same number of times as the number in the set).
- Use counting songs and fingerplays to practice counting forward, backward, by twos, by fives, by tens, etc.
- Using the brailler, have the student count spaces to get to the bell, starting from different points along the line; the student can also depress full cells to correspond with a particular number.
- Have the child count objects aloud as he or she individually drops them into containers; start by dropping one item at a time, then two at a time, and so on.
- Keeping track of game scores can be a motivating and relevant way of applying counting skills. For example, the child can count the number of points earned by individuals in a card game, or in a ball game.
- Record specific directions on tape for the student's independent practice. For example, using a tray with dividers, the student could place a certain number of items in the first section, a different number of items in the second section, and so on. Directions could also be provided on Language Master cards. Students could also be directed to place a card with the correct Nemeth Code symbol in each of the sections to correspond with the number of items.

- Have a “counting scavenger hunt.” Tell the child the location of several containers of objects (depending on the student’s memory, he or she could be given one location at a time, or several at once). The child must go to the locations, obtain the container of objects, count the number in the container, and then order the containers in correct number sequence. The student can then count up all the items for a grand total.
- Use the Nomad with overlays containing rows of tactual dots and shapes; program the Nomad to count shapes sequentially from left to right or top to bottom. The child touches shapes in the correct sequence and receives reinforcement as to the number place in sequence. The child must confirm the correct sequence of numbering (good for a student who has limited fine motor ability).
- The development of an autobiographical timeline (in cooperation with a student’s family) requires the student to actually plot significant events sequentially. This provides concrete reinforcement of number line concepts and the value and sequence of numbers, in a personally relevant and interesting format.
- Students can play a game called “Guess My Number” (Petreshene, 1985), to reinforce and practice the concepts of “greater than” and “less than.” Braille a numeral between 1 and 100 on a piece of paper without informing the student of the number. Ask the student to discover the secret numeral by asking “greater than” and “less than” questions, keeping track of the answers by recording them in braille. For example, if the teacher has chosen the number 19, the student might ask if the number is greater than 10; he or she would then record, “>10." The next question might be whether the number is 20; he or she would record “<20." Interim guidance can be provided if necessary; for example, the student could be told that now he or she knows the number is somewhere between 10 and 20. Roles can be switched, with the teacher guessing the student’s number. The score is kept by entering a tally for each guess; the person with the fewest tallies (guessing the secret numbers in the fewest attempts) wins the game.

Place Value

Activities for teaching place value

- Two students alternate drawing from a deck of braille playing cards. Each card is placed in one of 8 slots on a board, immediately after drawing. The goal is to create the largest 8 digit number. The students read their numbers after they are complete, and determine which is largest.
- “Wipeout” (Baggett, 1995) is a calculator game which can be tailored to the ability level of the student, using numbers with fewer digits for lower skill

levels. The student enters a number on a calculator; it contains a predetermined number of digits; each digit in the number must be different (e.g., 66 would not be permitted). The student is directed to "wipeout" the number, one digit at a time, by changing each number to zero as the teacher calls it out. For example, if the number is 68459, and the teacher calls out the number 5, the student has to subtract 50 in order to turn the 5 into zero. This game would be started with two digit numbers, increasing the digits as the student comprehends place value.

- "Number-in-a-Box" (Petreshene, 1985): State a number between 10 and 99. Braille the numeral and one example of a combination of numbers which equal its value (e.g., 43 = 3 tens and 13 ones); place this information in a box. Tell the student that there is a number in the box which is worth 43 ones; only ones and tens have been used to make the number. The student then brailles every combination of ones and tens to equal 43 ones until he or she guesses the combination which is in the box. Alternate teacher and student guessing numbers; keep score with a tally mark for every incorrect guess. The game ends when someone accumulates 25 tally marks or points; the object is to have the fewest points.

Resources for activities and materials

- Unifix cubes and towers provide students with concrete reinforcement of number value and one-to-one correspondence.
- Stern Structural Arithmetic cubes and rods, notched at each unit on each rod allowing for tactual comparison, can be effective aids in calculation problems.
- Focus (APH) is a program which provides nearly two hundred objectives and activities for teaching mathematics concepts and skills. A large portion of the program is devoted to basic concepts and early number skills.
- Workjobs I, Workjobs II and Workjobs for Parents (Educational Teaching Aids) provide numerous activities for early mathematics concepts and number skills.

Measurement

Use of measurement tools

It is critical to teach students about the availability and appropriate use of tools of measurement. Ideally, students should be exposed to a broad range of tools, including those which are not specifically produced for use by blind persons but can be modified, if necessary, in a cost effective, "low-tech" manner. Mainstream, unadapted tools should be used whenever possible, since they will ultimately be the least expensive and easiest to obtain; but certainly students should be aware of all that is available so they can make informed choices in the future.

Instruction should include background information regarding purpose of the tool, where or how to purchase it, and approximate cost. Specific instruction on proper use of the tool should be followed by multiple opportunities to use it in a functional application, either in the direct solution of mathematics problems or in activities of daily living.

Tools for various types of measurement

Following is a list of examples of tools which are either manufactured specifically for use by visually impaired people, or modified through easy, non-technical means (Whigham, M. & Utsinger, D., 1996).

Linear:

1. Braille rulers (variety of sizes and materials)
2. Braille Starett (12' tape, accurate to 1/8"; $25)
3. Click ruler (3', audible click every 1/16"; extensions are available; good for industrial arts; $75)
4. Micrometer (accurate to 5/100,000th"; $200; with voice module, $900)
5. Bernier Caliber (6", accurate to 100 mm; $200)
6. Tape measure with notches at each inch, staples at each foot

Liquid:

1. Battery operated level indicator ($20)
2. Long handled metal spoon, handle bent at 90 degrees to form dipper
3. Standard syringe with notches cut into handle and stop at end of plunger (farm supply store)
4. Standard plastic measuring containers with overflow holes punched out at certain levels

Weight:

1. Braille scales, for weighing up to 2 pounds
2. Talking scales, for weighing up to 10 pounds
3. Hobart scale, for weighing up to 30 pounds with accuracy within 1/100th of a pound; announces weight and price per pound ($1700)
4. Weight Talker III, for weighing heavy objects with accuracy to one pound ($75)
5. Balance scale with trays and tactile needle, for weighing liquids and small objects

Temperature:

1. Braille thermometer, for liquids

2. Talking thermometer, for liquids, air, and body ($125)

Time:

1. Braille clocks and watches
2. Talking clocks and watches

Activities for teaching measurement

- Cut several cardboard rectangles of different sizes (e.g., measuring 1" by 2", 1" by 3", 1" by 5", 2" by 3", etc.). Have the student measure the sides of each rectangle with a raised line or braille ruler. After explaining perimeter, have the student add the lengths of the sides and write down the perimeter of the rectangle. Next, give the student a box of 1" squares (wooden or plastic parquetry squares, or cardboard squares the teacher has cut). The student places 1" squares on top of the rectangle, determining how many squares are needed to completely cover the cardboard. After explaining that he or she has discovered the area of the rectangle (review the difference between perimeter and area), the student can be taught to multiply the length of the long side by the length of the short side to determine area. Answers can be compared with answers obtained using the 1 inch cardboard squares.
- The student can also place his or her rectangles along a line, according to size (area). He or she will see that rectangles of different sizes can have the same area (i.e., a 3" by 4" rectangle has the same area as one which is 6" by 2").
- Have the student draw a diagonal line with a crayon (or use a Wikki Stix to create a diagonal line) on some of the rectangles, and cut them into 2 right triangles. Have him or her measure the area, teaching that since each triangle is half of a rectangle, its area can be determined by dividing the area of the rectangle in half.

BASIC NUMBER FACTS AND OPERATIONS

It is very important that students see mathematics, and the calculations they perform, as part of their daily life. Providing opportunities to apply basic concepts and operations in daily activities will reinforce students' skills and motivate them to progress in mathematics. They can use addition to figure total amounts of toys or snacks, and to keep track of their bank accounts or team equipment. Students can use subtraction to make comparisons between what they have and what they need for a game or other activity, to budget, and to calculate remaining items as they are used, or to calculate change when a purchase is made. They can multiply to figure larger totals, and to transform units from one measure into another. They can divide to determine equal portions of items, or to figure daily averages for sports scores or percent scores for quizzes or games.

In order for students to calculate using these four basic operations, they must first have developed basic concepts (including more, less, many, etc.), one to one correspondence, the concept of sets, and basic number sense. As students begin to learn to calculate, the following teaching considerations should help:

- Emphasize concept development rather than process or rote memorization.
- Apply operations to real life situations which are of interest to the student (e.g., provide opportunities for students to determine quantities of materials needed to play a game or complete a project and to estimate the price to purchase these materials). At first, provide examples for the student, then ask the student to provide his or her own examples which he or she sees as relevant uses of different operations.
- When students are using manipulatives, encourage them to search the entire "field" to make sure they are aware of all the objects with which they must work. Using trays or mats can help to identify this field and the area they must search.
- Word problems are very effective since they involve practical application of skills. To assist students in developing the skills necessary to solve word problems, it may be helpful to provide a problem solving model. First, identify the specific kinds of information needed in a particular problem; then provide two or three choices of operation statements to solve the problem. Eventually, students will be able to identify appropriate operations independently.
- Teach the concept of complements or partners for addition, subtraction, multiplication and division. For example, the number 5 is made up of 2 and 3, 1 and 4; 24 is made up of the factors 8 and 3, or 2 and 12, etc. This concept not only increases the student's ease with number facts; it also facilitates mental mathematics.

- When teaching facts, focus on 2 or 3 related facts at a time. Emphasize accuracy first, then speed. Maintain a chart of mastered facts to help the student recognize progress.
- Small flip charts can be provided at a student's desk, with cues for steps in particular types of problems as a reference or reminder.

Activities for teaching basic operations

- The number line (APH) can be used for working on operations, relationships, fractions, and decimals. Number lines are especially useful when they are stretched across the top of the student's desk, less helpful when confined to the dimensions of a braille page. The number line attached to the student's desk is particularly helpful when the student is using the braillewriter. Students can find the larger number, count forward for addition and count backward for subtraction. Students can use number lines for working on positive and negative numbers as well. Thermometers can also be useful in teaching positive and negative numbers.
- Flashcards can be used along with manipulatives for working on the basic operation facts and the Nemeth Code; later, students can braille out their own mathematical sentences.
- Students having difficulty learning addition, subtraction, multiplication or division facts can make cards of those facts with which they are having the most trouble. Write the problem on one side of the card and the answer on the reverse side. Teachers can use creativity and relevant examples, scratch and sniff stickers, etc. to make these teaching aids interesting. Many models of these types of aids can be found in teacher stores.
- To practice mathematics facts, students can roll dice, then add, subtract, multiply or divide the numbers thrown. A variety of rules can make this into a game. For example, the first student whose cumulative numbers add up to 100 wins, or students start from 100 and subtract numbers thrown and the first one reaching zero wins, or students multiply the two numbers thrown and then add these numbers cumulatively. Students could also rename fractions formed by combining the two numbers thrown.
- "Start Here...End There!" (Petreshene, 1985): give the student a paper with a pair of numbers brailled on the first line. The student then determines at least 3 ways to get from the first number to the second, brailling each series of computations below the number pair. He or she may use any combination of addition, subtraction, multiplication, and division. For example, if the designated numbers are 3 and 24, some possibilities would be:

3x8 = 24
3x9–3 = 24
3+10+10+1 = 24

- Mad Minute, available on disk, can be a motivating way for students to practice basic operations.

Suggestions for teaching addition and subtraction

While the above tips relate to teaching any of the basic arithmetic facts, the following suggestions could be especially appropriate for working on addition and subtraction:

- For students who have great difficulty remembering number facts, an additive principle may be helpful. For example, teach the "doubles" (2+2, 3+3) for all facts up to 10. These can serve as main facts from which other facts may be derived (as in the problem "2+2+1" or "one less than 4+4").
- Magnets can be used on a small cookie sheet or magnet board to form groups for addition or subtraction problems. Unlike the manipulatives usually found in primary level classrooms, individual magnets can be moved easily without falling off the surface. At a more advanced level, a cookie sheet or magnet board could be used as a personal "chalkboard" where individual tiles labeled with Nemeth Code numbers and signs of operation (and affixed to magnetic tape) could be arranged to form a variety of problems. Wikki Stix could be used for separation lines. The student could work on a variety of problems at his or her desk while the teacher works the problems at the board. Classroom teachers would have to cooperate by verbalizing the problems clearly!
- The work tray from SAVI or APH can serve as a good organizer for making simple addition and subtraction statements. For example, a collection of small manipulatives can be placed in the larger section on the left of the tray. Four of these objects could be placed in the first of the three smaller sections and 2 more placed in the second section. The student could then take the objects from both sections and place them in the third section, counting them for the total of 6. The same procedure could be used for subtraction statements. Problems involving the addition or subtraction of zeros could also be worked out in a very concrete manner using this approach.
- To teach addition to young children, cubes that attach to each other (e.g., Unifix cubes) can be an effective aid. The student can be given a specific number of cubes, and asked to count them; then the student can be given a second group of cubes which are counted and attached to the first group. The total number of cubes can then be counted. Beads on a string can also be used by giving the child a string with first 2 beads of one shape (circles),

then 3 beads of a different shape (squares). The student reads the problem from left to right (2+3=5). Subtraction could also be practiced by presenting the combined amounts first, and then having the student remove the particular number of cubes or beads (5-3=2).

- When setting up spatial arrangements involving regrouping (i.e., carrying and borrowing) or cancellation, a frame with 9 cubes arranged in a vertical line could be used as an aid. As the student fills the frame with counters, he or she can see that if another place is needed for a sum larger than the 9 available, he or she must place it above the frame. A separated section above the frame could be added to hold the carryover number.
- Students can work on the concepts of "greater than" and "less than" while they work on addition and subtraction. For example, two numbers (8,13) could be given to the student in a particular sequence, and the student would have to state the relationship of the first number to the second (8 is less than 13). Next, the student could use the operations of addition or subtraction to solve problems (e.g., how much less is 8 than 13? What number must we add to 8 to make it equal to 13? What number must we add to 8 to make it greater than 13?).
- The game "Mystery Coins" (Petreshene, 1985) provides practice for basic mathematics facts and money skills. Place a variety of coins in a paper bag, and announce the total value and number of coins (e.g., 35 cents; 8 coins). The student guesses which exact coins are in the bag. When he or she states the correct answer (in this example, it is 3 dimes and 5 pennies), he or she counts the actual coins for verification.

Suggestions for teaching multiplication and division

Following are several suggestions that might be especially helpful for teaching multiplication and division:

- Remind students of these "tricks" that can help them to remember multiplication facts:

  Zero times any number is zero.

  One times any number is the number itself.

  Nines are "magical": the answer to the nines facts always add up to nine (e.g., 9x8=72; 7+2=9).

- Construct a list with the numbers 0-9 in the left column, and the numbers 9-0 in the right column. The resulting number combinations will be the answers to the facts from 9x1 to 9x10:

  x1: 0 9

  x2: 1 8

x3: 2 7
x4: 3 6
x5: 4 5
x6: 5 4
x7: 6 3
x8: 7 2
x9: 8 1
x10: 9 0

- Start with an additive approach, using groups of manipulatives (e.g., 2+2+2= 3x2, 4+4+4+4=4x4, etc.).
- Stress the associative principle, having students make and remake statements regarding different arrangements of objects (e.g., using an egg carton, students can see that 6x2=12 and that 2x6=12).
- Students can use card games with factors (3x8) on one card and products (24) on another to make a matching pair.
- When multiplying different numbers by the factor 9, students can use the "finger trick". First, the student places both hands out in front of himself or herself, palms down, and counts the fingers as 1-10 starting with the little finger of the left hand. Then the student folds under the finger corresponding to the multiplier of nine, and reads the product by reading the number of fingers before the finger folded under, followed by the number of fingers following the folded finger. For example, in the problem 4x9, the student would fold under the index finger of the left hand because that is the 4th finger in the sequence. The student would then read the answer as 36 because there are 3 fingers before the folded finger, and 6 fingers after the folded finger.
- Students may find "multi-blocks" useful. All the relevant multipliers, multiplicands and products are labeled in Nemeth Code on the different sides of the blocks. Students can make appropriate matches by arranging the blocks so that combinations of factors and products face up.
- To practice both muliplication facts and the associative concept, students can play a tic-tac-toe game. Each square of the tic-tac-toe board contains a multiplication problem without the answer; factors should be presented in both orders but in different squares (6x8, 8x6). Students must find 3 combinations in a line which yield the same product. This activity could also be used for practicing addition facts, or other combinations of basic facts.

Fractions and Decimals

Grasping the fundamental concepts underlying the addition, subtraction, multiplication, and division of fractions is difficult for many students. Not being able to

visualize the concepts as they may be depicted in pictures makes the understanding of these fundamental operations even more difficult. One can train students to readily calculate the correct answers to problems containing fractions, but being able to complete the operations correctly does not guarantee that the student has a fundamental understanding of the meaning of the four operations as they pertain to fractions. What does it mean to add 3/4 and 1/2? What does it mean to subtract 1/2 from 3/4? What is the basic meaning of the answer to the problem, 3/4 times 1/2? What does it really mean to divide 3/4 by 1/2? The use of manipulatives to illustrate the basic operations in which fractions are involved will enhance understanding by the blind learner. The following illustrate that point.

Teaching operations with fractions

In the problem, 3/4 added to 1/2, the teacher might use two cardboard circles divided into fourths. Three pieces from one circle represent 3/4. Two pieces from the other circle represent 1/2. Bringing the three pieces (3/4) together with the two other pieces (1/2) results in a total of five pieces (5/4). The pieces can be re-assembled into one whole circle comprised of four pieces, with an additional single piece representing 1/4. The answer, thus, is 1 1/4. The manipulation of the pieces provides concrete evidence of the concept of adding fractions.

In subtracting 2 1/2 from 3/4, a similar activity would be effective. In this case, three pieces would be displayed. To subtract 1/2 from 3/4, one would simply remove two of the pieces, representing 2/4 (1/2). The remaining piece, then, is the answer: 1/4.

Multiplication of fractions is a somewhat more difficult operation to understand. If 3/4 and 1/2 are multiplied, the result is 3/8. Multiplying 1/2 times 3/4 actually is asking the question, what is 1/2 of 3/4? To illustrate this point, a cardboard circle is divided into eighths. To convert the fraction 3/4 to 6/8, 6 pieces of the circle are used, each of which constitutes 1/8 of the circle. One-half of 6 is 3. Thus, multiplying 1/2 times 3/4 is 3/8.

The basic meaning of division of fractions is even more difficult to comprehend. If one divides 3/4 by 1/2, the result is 1 1/2. This is the way one would determine how many 1/2's are in 3/4. Once again, the teacher can display a partial circle with three 1/4 pieces. Two of these 1/4 pieces are removed, representing one 1/2. The remaining piece represents 1/4. The fraction 1/4 is one-half of 1/2; thus, there are one and one-half 1/2's in 3/4.

Strategies for teaching operations with fractions

The following strategies can facilitate the student's working with fractions:

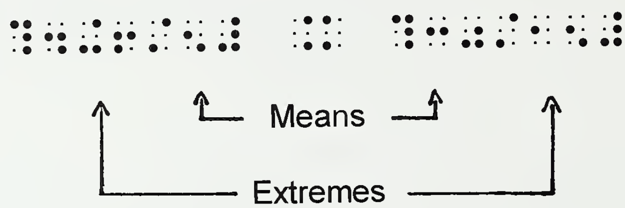
- The student should be instructed that what is between the opening fraction indicator (dots 1-4-5-6) and the fraction line (dots 3-4) is the entire numera-

tor, and what is between the fraction line and the closing fraction indicator (dots 3-4-5-6) is the entire denominator.

- When teaching order of operations, students need to learn that all computations above the horizontal fraction line are to be completed, all computations below the horizontal fraction line are to be completed; then division of the numerator by the denominator can be carried out.
- It may be helpful to teach the student "means" and "extremes." In braille, the extremes are the farthest left and farthest right portions of the problem. For example, in the following problem:

$$\frac{4+6}{5} = \frac{30}{15}$$

when written in braille on one horizontal line, the extremes are the numbers written to the farthest left and farthest right (4+6, 15); while the means are written on the inside right and left (5, 30).



The language of fractions

Abraham Nemeth (1996) recommends use of the following language in order to communicate with clarity when reading mathematics problems containing fractions to a blind student:

> A simple fraction (which has no subsidiary fractions) is said to be of order zero . . . A fraction of order 1 is frequently referred to as a complex fraction, and one of order 2 as a hypercomplex fraction. Complex fractions are fairly common, hypercomplex fractions are rare, and fractions of higher order are practically non-existent. The order of a fraction is readily determined by a simple visual inspection, so that the sighted reader forms an immediate mental orientation to the nature of the notation with which he is dealing. It is important for a braille reader to have this same information at the same time that it is available to the sighted reader. Without this information, the braille reader may discover that he is dealing with a fraction whose order is higher than he expected, and may have to reformulate his thinking accordingly long after he has become aware of the outer fractions.

ADVANCED MATHEMATICS

The study of algebra, geometry and other advanced mathematics topics by blind individuals presents several challenges. One of the most serious of these is the fact that blind students cannot visualize the graphical representations of complex mathematical concepts which aid sighted students in their understanding of those concepts. An obvious example of this problem is the difficulty in representing three dimensional objects which may be the focus in the study of certain advanced topics such as geometry. However, even basic concepts often taken for granted can present difficulty for the blind student as he or she pursues more advanced study. Bev Weiland, a programmer-analyst at the University of Delaware, illustrates this in her comment:

> I have been totally blind since birth and have studied algebra, geometry, and calculus. I found geometry especially difficult because I lacked the understanding of many spatial concepts ... I found that I had difficulty understanding such concepts as how four walls meet the ceiling, and I actually stood on a chair to study this (Dick & Kubiak, 1997, p.344).

Specific issues and guidelines related to the facilitation of the blind student's use of tactile displays and graphics to aid in such study is covered elsewhere in this manual, and the reader is referred to that section.

A second problem relates to the fact that the blind student cannot visualize complex mathematical expressions in their entireties. He or she must view these expressions in small portions rather than all at once as their sighted peers do. This inability to view mathematical expressions in their entirety requires the blind student to retain large portions of the expression in his or her memory as it is manipulated. This can make the study of algebra and other advanced topics more difficult for the less capable blind student than it is for his or her sighted peer with equal cognitive ability.

Techniques which facilitate students' abilities to calculate and remember mathematics facts mentally, and to use tools for keeping track of partial sums and products, can be very helpful in this regard, although they will not eliminate the problem.

As with the development of basic concepts, the need for concrete experiences is critical to the development of many advanced mathematical concepts for the blind student. Acting out story problems and applying these problems to everyday situations is just as important at the advanced levels of mathematics as it is at basic levels, if students are to become capable of using their mathematics skills in functional situations or as a foundation in their pursuit of even more advanced mathematical and scientific learning. The use of models, manipulatives, and real life items found in everyday classrooms and living environments also plays an important role in providing support to the development of mathematical skills and concepts at all levels. These

items can be used for functional measuring, comparing, deduction and induction, as well as for motivating students to solve relevant problems. The following strategies are intended as illustrative examples of the use of concrete experiences to support higher mathematics concepts. They are included here as representative samples only; teachers are encouraged to develop a multitude of concrete experiences and problem-solving situations to help students understand the concepts involved in higher mathematics.

Strategies for teaching concepts in higher mathematics

- Many students are confused by the signs of inequality (< and >), and inappropriately apply one of them when the other is needed. Since the symbols for each of these is a two-celled symbol, he or she needs only to remember that the cell containing only one braille dot is always pointing to the value which is less than the other. For example, in “nine is greater than four” (9>4),

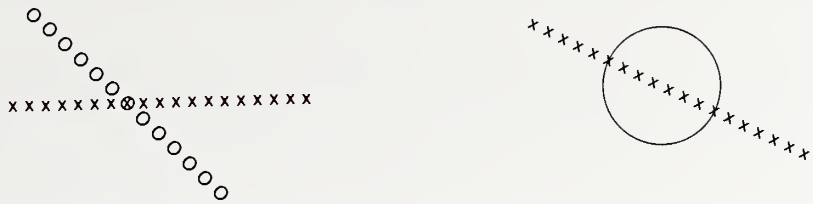
  the single dot 2 is pointing toward the lesser value, four. Likewise, in “four is less than nine” (4<9),

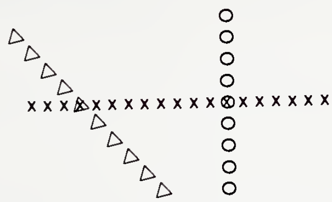
  the single dot 5 is still pointing toward the lesser value, four.

- When using a braille keypad, each sign of comparison begins with a configuration formed with the right hand.
- Each of these symbols in braille retains one cell of the two cells of the braille symbol for equality. This can be used to demonstrate that the compared relationship is skewed in favor of one direction. The braille symbol for equality uses two braille cells that are symmetrical (dots 4-6, 1-3), just as the left and right side of an equation are symmetrically balanced.
- When presenting the braille symbols and teaching the concepts of intersection (dots 4-6, 1-4-6) and union (dots 4-6, 3-4-6), note that the braille “plus” sign (dots 3-4-6) is part of the symbol for union. This is because the new set which results from combining two original sets is similar to the sum in an addition problem.
- Print users often compare the symbol for union to an uppercase letter “U”. A comparison for a braille student would be to notice that the dot configuration for the braille letter “U” is the first cell of the symbol, and the reversed image of the braille letter “U” is the second cell. This might be confusing, however, for a student with reversal problems.
- To teach the concept of intersection to a group of students, have the students form two lines. Refer to each line by a different name. Have the

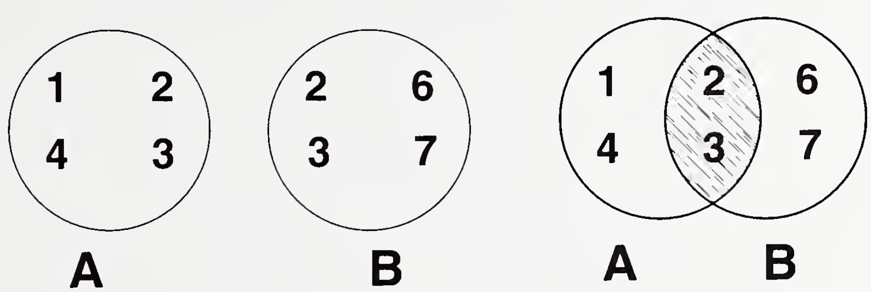
two lines march into a pattern as shown below. The student who is in both lines is a member of both lines, and is therefore the intersection. This concept can later presented with the use of raised line diagrams.



More than one intersection can also be illustrated as such:



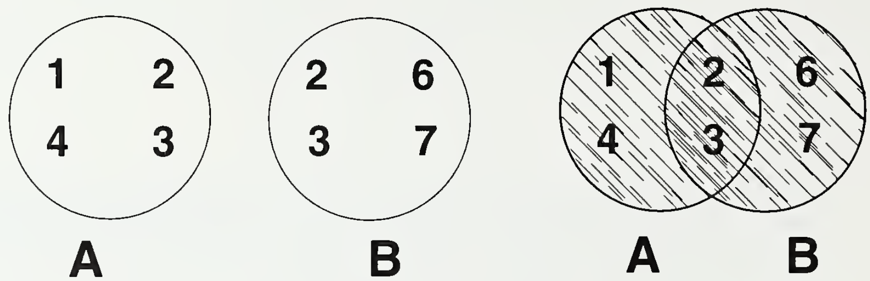
- To teach the concept of intersection at a more symbolic level, the teacher can use two rings about 8 inches across, with cellophane stretched across one frame and a very thin fabric across the other. Label one ring A and the other B. Have the student examine each frame separately, then slide one over the other so that only a portion overlaps. Clip the frames together so that the student can now examine the area which overlaps. This area is the intersection. The student can express this with the statement $A \cap B$. The teacher can later place braille labels on each of the materials covering the frames; two of the labels should be duplicated on both frames. The duplicated labels should be positioned toward the edge of the frame so that, when the frames are partially overlapped, these duplicated labels will be included in the overlapped portion. A raised line drawing of circle A, circle B, and the partial overlapping of A and B can later be presented to the student for examination.



- To demonstrate the concept of union, the student can place several items in each of two boxes (e.g., Box A contains a stylus, pen and paper clip, while Box B contains a safety pin, key ring and slate). The student then empties

the contents of both boxes into a third box or into a tray. The result is the union of the two boxes. Later, duplicate items can be included, but when the two boxes are combined, all duplicate items must be removed before identifying the elements forming the union.

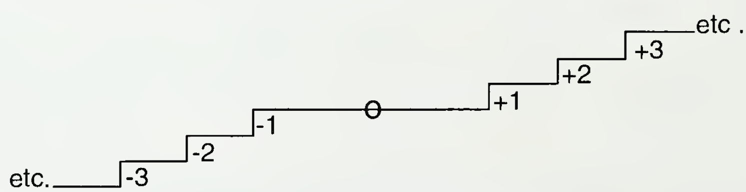
- At a more symbolic level, the rings A and B described above can be examined, first separately, then again in the partially overlapped position. This time, however, the union is represented by all the area covered by either or both the frames. If the rings are used with the labels, then the union includes all the labels on either or both of the overlapped frames, minus any duplicate labels that may be present. The student can express this with the statement A U B. Later, a raised line drawing can be used to represent this concept.



- A mnemonic device for remembering factor polynomials is FOIL:

  F=first
  O=outside term
  I=inside term
  L=last term

- A "number line" made of stairs can be used to teach the concept of signed numbers. Take the students to a landing between floors, with stairs going up and down. The landing is zero. The stairs going up from the landing are positive numbers, while the stairs going down are negative numbers.



Have the student go to "positive seven" (+7), or seven steps up. Then ask the student to add a "negative nine". To do this, ask the student which way is negative? When the student responds "down", ask the student to move down nine steps, and to tell where he is in relation to the landing [he is at "negative two" (-2)]. The student can relate what he has experienced with

the number sentence “+7+-9=-2. This process can be continued with additional addends of both positive and negative value.

- In algebra, to apply the concept of equality as being two amounts which are the same, use a balance scale which can hold two dishes (e.g., the scale from SAVI). Have the left dish represent the left side of an equation while the right dish represents the right side of the equation. If the scale has a needle, it must be perpendicular to show equality (serving as the equal sign). If the scale does not have a needle, the fulcrum can serve as the equal sign.

  Have the student place a weight in one dish and try to balance it by placing an equivalent weight in the other. Students could start with a 2 gram weight in each dish; later they could place a 10 gram weight in the left dish, and use smaller weights in the right dish to balance. Students can later add and subtract equal values to the two balanced dishes.

- To learn about combining like terms, students can use what they already know about place value. The teacher can make a place value chart applicable to algebra by using library pockets arranged in columns, and labelled as below in braille. A set of cards can then be brailled with values for each column: 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +0x, +1x, +2x, +3x, +4x, +5x, +6x, +7x, +8x, +9x, $+0x^2$, $+1x^2$, $+2x^2$, etc. On the reverse side of each card, the same value should be brailled with a negative sign.

  First, students can review place values and their application to algebra by going over the chart. The teacher should stress that values in a column may only be combined with values in the same column, and that their “value” is formed by multiplying the place’s value by the amount in that place.

etc.	$+x^3$	$+x^2$	+x	+numbers	$+x^{-1}$
etc.	$+10^3$	$+10^2$	$+10^1$	$+10^0$	$+10^{-1}$
etc.	+1000	+100	+10	+1	+1/10
etc.	+thousands	+hundreds	+tens	+units	+tenths

Just as place values can accommodate decimals (fractional values with powers of ten as the denominators), this chart can be expanded to include variables raised to negative exponents, using pattern analysis to continue the series with ascending negative exponents (e.g., $+x^2$, x , etc.).

The student can arrange the brailled cards to work specified problems. As they work blems, they combine like terms by collecting only the cards in one column at a time, adding them together, and recording the results in descending order of the power of the variable (e.g., $17x^2$-6x +2y-9).

Example: Addition

$$\begin{array}{r@{\quad}r} & 715 \\ + & \underline{263} \end{array}$$

The 5 and the 3 are in the units column; their sum (8) is multiplied by the place value (1) with a result of 8.

The 1 and the 6 are in the tens column; their sum (7) is multiplied by the place value (10) with a result of 70.

The 7 and the 2 are in the hundreds column; their sum (9) is multiplied by the place value (100) with a result of 900.

The result of each column is added together (8+70+900) with a result of 978.

Example: Algebra

$$\begin{array}{r@{\quad}l} & 7x^2+1x+5 \\ + & \underline{2x^2+6x+3} \end{array}$$

The 5 and the 3 are in the “numbers” column; their sum (+8) is multiplied by (1) with a result of +8.

The 1 and the 6 are in the “x” column; their sum (+7) is multiplied by (x) with a result of +7x.

The 2 and the 7 are in the “x^2“ column; their sum (+9) is multiplied by x^2 with a result of $+9x^2$.

The results of each column are added together ($9x^2$+7x+8).

- To develop the concept that variables represent unknown amounts, students can use a balance scale which holds two dishes (e.g., SAVI). The

teacher brailles a variable ("y") on a light weight bag, places a gram weight in the bag, and seals it shut (the student does not know the weight placed in the bag). The bag, along with one or more other weights (e.g., 7 grams) are placed into one dish, while enough weights to balance (e.g., 37 grams) are placed in the other dish.

To observe the process of isolating variables, the student must remove the same number of gram weights from both sides of the scale until only the bag is isolated in one dish. By deduction, the student can determine the amount of grams that are in the bag by counting the weights in the other dish which balances it. The student can then open the bag and weigh the weight to confirm its value.

The student can then braille the process he or she has carried out in an algebraic equation.

Example: $\underline{[y]+7 = 37}$ $\underline{[y]+7-7 = 37-7}$ $\underline{[y] = 30}$ (weight of bag)

^ ^ ^

- To teach the concept of the distributive property for X and demonstrate how a term outside parentheses is distributed to each term inside parentheses, teachers can use a painting activity. First, the teacher labels a box with braille mathematics parentheses (dots 1-2-3-5-6 and 2-3-4-5-6). The teacher then places a selection of items (e.g., an eraser, paper cup, ruler) in the box, with each item labelled as a variable (x for the eraser, +7x for the cup, -12 for the ruler). Then the student can work the problem

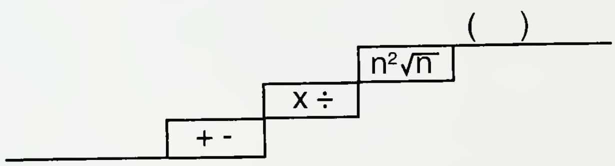
$$2x(x^2+7-12)$$

The student is directed to brush all the items in the box with water (represented by 2x), and then leave the items out of the box "to dry". When asked how the items outside the box are different from when they were in the box, the student can identify that now all the items are wet and that each item was altered by the water. In the same manner, all terms (represented by the items) are multiplied by the term (represented by the water) outside the parentheses (represented by the box).

Therefore, as the student applies water to each item,

$2x(x^2) = +2x^3$
$2x(+7x) = +14x^2$
$2x(-12) = -24x$
Arranged horizontally: $2x^3+14x^2-24x$

- To observe that angles are not affected by the length of their rays, students can place items such as the long cane perpendicular to the floor and use their braille protractor to measure the right angle formed, noting that one ray (the cane) is much shorter than the other (floor). Students can also use differing lengths of yarn and align them as rays to form specified angles.
- To demonstrate the concept of correct movement of the decimal point in metric, the student can use a paper plate as a “dancing decimal point.” A group of students can stand in a row, each with an assigned number. The paper plate decimal point is moved between each student, either to the left or to the right, depending on whether there is a change to a larger metric unit or to a smaller metric unit. For example, students could be named with each of the following digits: 2, 5, 9. To have the number represent 259 meters, the decimal point plate can be placed to the right of the 9. Then, to change to kilometers, the decimal point plate is moved three places to the left, before the 2, representing 259 kilometers, and so on. If there are not enough students, lined up chairs could also be used.
- To teach the concept of hierarchal mathematics operations, the teacher can use a set of steps made with boxes, arranged as below, with one side open so that the “rules” for each step or level can be inserted. For example,



Using these steps, students can perform operations in a problem in the correct sequence.

Example:	7+4x(8-3)	Highest operation involves parentheses; combine terms
		There are no roots or powers; move down a level
	7+4x5	Next level is multiplication; perform operation
	7+20	Next level is addition; perform operation
	27	

Example:	$8-25\div\sqrt{25}$	Highest operation involves square root; perform
	8-25÷5	Next level is division; perform operation
	8-5	Next level is subtraction; perform operation
	3	

If a problem has more than one of the same level of operations (e.g., a division and a multiplication operation), the operations should be performed starting from the left side of the problem.

Example: 5+6÷2x4
5+3x4
5+12
17

- To remind students that rules which apply to one level of an operation do not necessarily apply to another, short "help" cards can be brailled and placed in the hierarchal boxes described above, or in a notebook for quick reference. For example,

Rules applying to fractions include:

For addition/subtraction, the denominators must be the same; add the numerators

For multiplication, multiply numerators across; multiply denominators across

Rules applying to directed (signed) numbers (when working with only 2 numbers at a time) include:

For addition, if the signs are the same, add the values; the sum gets the sign of the original numbers. If the signs are different, subtract the values; the difference gets the sign of the larger (absolute value) original number.

For multiplication and division, like signs (+x+, -x-) result in a positive number, while unlike signs (+x-, -x+) result in a negative number.

TEACHING NEMETH CODE

Approach

The following paragraphs describe in detail a recommended approach to teaching the braille mathematics symbols along with suggested alterations in format. Generally, strict adherence should be maintained in teaching the structure and function of the braille mathematics symbols. Materials and content should conform with the commonly accepted rules for writing the braille mathematics symbols. If a teacher makes changes in the written forms of the symbols, considerable confusion will arise when students read properly transcribed mathematics. Therefore, no matter how awkward some of the symbols may be to write, their proper form should be maintained.

There is one area of braille mathematics in which it is highly recommended that changes should be made in both the form of the symbols and format. This should be done when the braillewriter is being used as a tool to calculate addition, subtraction, multiplication, or division problems which are spatially arranged (as explained in detail in the section titled, *Calculation Tools and Aids)*. If one were to rigidly heed transcription rules when using the braillewriter for this purpose, the use of the braillewriter as a calculation tool, which is difficult at best, would be much more difficult than it needs to be.

One must bear in mind the goal at hand as the student studies mathematics. In the case of performing arithmetic calculations, the goal should be to learn and practice the basic skills involved in the fundamental arithmetic operations. If strict adherence to the rules for brailling the Nemeth Code interferes with the achievement of that goal, then modifications in the rules should be made. The student should be encouraged to follow the Nemeth Code rules, however, whenever brailling any area of mathematics, including algebra, geometry, trigonometry, and calculus, if the braillewriter is not being used as a calculation tool.

The mechanics of teaching Nemeth Code symbols are similar to those which are employed in teaching literary code. However, no research exists regarding the proper sequence in which to introduce Nemeth Code symbols. It is, therefore, recommended that the Nemeth Code symbols should be introduced as they occur in the print versions of the mathematics texts which are being used in the program in which the blind student is enrolled. If the student is not thoroughly familiar with the symbols at the time that the mathematical concepts are being studied, he or she has no way to write those symbols and thus to study the concepts.

The student should be taught the correct meanings of the symbols as well as the rules governing the symbols, particularly those which do not have print equivalents. These include, but are not limited to, such symbols as the baseline indicator (dot 5), the superscript indicator (dots 4-5), the subscript indicator (dots 5-6), the opening fraction indicator (dots 1-4-5-6), and the closing fraction indicator (dots 3-4-5-6). In

order to truly understand the code of braille mathematics, the student must have a full understanding of the rules governing the use of the symbols which comprise the code.

Students should always be presented with virtually flawless braille mathematics. This is even more important in braille mathematics than it is in the literary code. When one reads material written in the literary code which contains errors, the correct meaning can usually be determined through the use of context clues. There are no context clues available, however, when reading mathematical symbols. It is therefore very important that the braille mathematics symbols are precise and accurate.

Strategies for Teaching Symbols and Rules

- The contraction for "in" (dots 3-4) cannot be used with the abbreviation for "inch" or "inches" (as in "in."). The abbreviation "st" may only be used with "street" or "saint" and not with any other word ("st" for "straight" must be brailled in uncontracted form using two letters).

  The following rhyme is an example of a mnemonic device that may help students recall these contraction restrictions:

  It is not a horrid sin
  To abbreviate the "i-n" in "min."
  If this rule you have forgot,
  It's "in" for "inches" you may not.
  Know these rules, you'll be a "St.",
  And likely make your teachers faint.
  The other time that "s-t" meet
  Is when they cross a busy "st."

- Students often confuse the numeral one (dot 2), and the baseline indicator (dot 5). Using repetitive exercises often helps students to read dots carefully, and distinguish between similar dots. Braille worksheets which incorporate the multipurpose indicator and the numeral one, often using exponents followed by baseline characters, provide practice in distinguishing these symbols. Two examples of such problems are:

$$7x^2-1$$

$$10^{11}+5$$

CALCULATION TOOLS AND AIDS

According to the *National Council of Teachers of Mathematics*, all students need meaningful mathematics with appropriate tools. It is particularly important for students with visual impairments to be exposed to the use of multiple mathematics strategies and tools, including use of the abacus, braillewriter, mental mathematics, fingermath, talking calculators, and the calculator function of electronic notetaking devices. These strategies and tools should be introduced early in a child's educational program, and continually reinforced throughout the school years.

Sequence

This section deals with the use as well as recommended sequence of instruction of the braillewriter, the abacus, and the talking calculator as tools for arithmetic calculation. Each of the three tools has specific advantages and disadvantages which will be discussed. In addition, suggestions will be made for integrating the three tools in the overall mathematics instruction program.

Of the three tools, use of the braillewriter is the most time-consuming and cumbersome. The braillewriter is extremely slow and awkward to use as a calculation tool. Using it for this purpose is analogous to doing mathematical calculations with a typewriter by a sighted individual. Under these circumstances, the reader may dispute the advisability of expending the time and effort needed to teach the use of the braillewriter as a calculation tool. Nevertheless, blind students benefit from knowing the steps which are required to carry out arithmetic calculations in this manner. In general, while it should not receive heavy emphasis and should be combined with the use of other tools, the braillewriter as a calculation tool should be introduced at the beginning of mathematics instruction.

Use of the braillewriter emulates how sighted individuals perform arithmetic calculations. Although it is not absolutely necessary for blind persons to perform calculations in the same manner as sighted individuals in order to be successful, it is beneficial to the blind learner to know the processes which are necessary to perform general arithmetic calculations. For true understanding of the procedures for carrying out arithmetic calculations, and thorough comprehension of basic mathematics, the braillewriter is the only device available to blind students. It is the only way for a blind student to write the steps involved in carrying out an arithmetic operation. In the past, arithmetic slates such as the cubarithm and the Taylor slate were advocated for this purpose. These are no longer used in up-to-date programs for blind students because experience has shown that these tools are very cumbersome and inefficient when compared to the use of the braillewriter.

The Braillewriter

Steps should be taken to make calculation procedures with the braillewriter as easily accomplished as possible. Alterations in the symbol forms and format are needed to enhance the effectiveness of the braillewriter as a calculation tool. The recommended changes in methods used to perform arithmetic calculations, as well as modification of the strict rules for transcribing Nemeth Code, make it easier for the student to carry out the necessary steps to add, subtract, multiply, and divide with the braillewriter. The following approach should be used in carrying out the four basic operations with whole numbers, decimals, and fractions.

Whole numbers

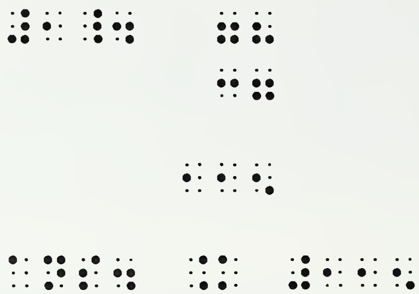
Addition of whole numbers

A mathematics problem will be copied from the student's textbook, which has been transcribed into braille. The item identifier (number of the problem) should be written following Nemeth Code rules, beginning with the numeric indicator (dots 3-4-5-6), the Nemeth Code number, followed by the punctuation indicator (dots 4-5-6), and then the period. Two spaces are left between the period and the first addend (the first number in the list of numbers). No numeric indicators should be placed before any of the numbers in the problem, including the answer.

Immediately below the first addend, subsequent addends should be brailled. These numbers should be placed in the proper columns, as they would be in print. A blank line should be left under the last addend. The horizontal line which separates the addends from the answer (a line of dots 2-5) should not be used; neither should the sign of operation (e.g., plus sign) be brailled, since it is clear from the written exercise that this is an addition problem. The principle to follow is to braille as little as possible in order to save time. On the other hand, sufficient symbols must be brailled to make the steps in the operation clear. To leave a blank line below the last addend, the paper advance control should be tapped twice after the last addend has been brailled. The carriage of the braillewriter should then be positioned in the units column. The student can use his or her finger to make certain that he or she feels the units column being "pointed to" by the carriage.

After the carriage of the braillewriter has been positioned correctly, the calculation can begin. The student simply reads the answers in the proper column, adding them as they are read. The units number of the answer is written, and the student backspaces twice to position the carriage in the tens column. If there is a number to be regrouped (carried over), the student must remember that value. He or she then adds that value to the number at the beginning of the tens column and proceeds down the column, adding the numbers as they appear under his or her finger. If the student experiences difficulty holding the regrouped number in his or her memory, an abacus can be used to temporarily store it.

The student then writes the number and backspaces twice. This procedure is repeated until the operation is complete. The teacher may wish to have the student clarify which number is the final answer by having the student rewrite it below the problem, noted as follows: ans. = the number, with a numeric indicator preceding it.



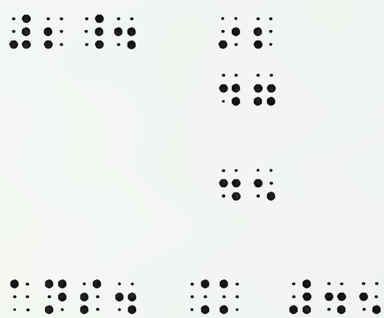
#1. 78
37

115

ans. = #115

Subtraction of whole numbers

The method for subtraction of whole numbers follows the same approach as outlined above with one obvious difference: the numbers are subtracted rather than added! Once again, no numeric indicators are used in the problem. The minus sign is not used. A blank line is substituted for the usual line of dots 2-5. The carriage is positioned in the proper column. The subtraction is performed and the number is written in the correct position. The backspace key is tapped twice, positioning the carriage in the next column to the left. If regrouping (borrowing) is necessary, the student is required to remember that he or she had performed that portion of the operation. As in addition, the writing of regrouped numbers with the braillewriter is too complex to make the operation useful. Once again, if the student has difficulty remembering these numbers, an abacus can be used to temporarily store them.



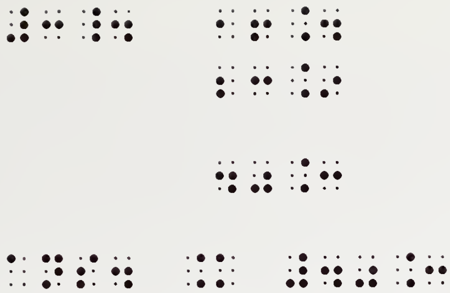
#2. 92
47

45

ans. = #45

Addition and subtraction of numbers with decimals

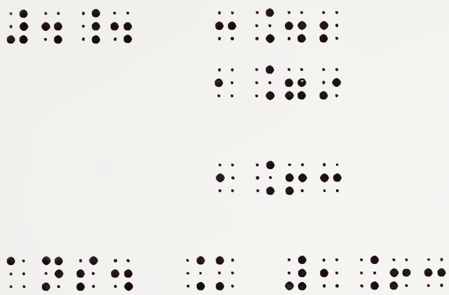
The procedure required for addition and subtraction of numbers with decimals is similar to those described above. The decimal point (dots 4-6) must be positioned correctly in the problem, following the conventions established in printed mathematics. No other variances from the above procedures are required.



#3. 16.4
23.9

40.3

ans. = #40.3



#4. 3.42
1.79

1.63

ans. = #1.63

Multiplication of whole numbers

The procedures involved in multiplication will be presented using two different types of problems. The first involves a two-digit multiplicand and a one-digit multiplier. The second involves a two-digit multiplicand and a two-digit multiplier. The methods for multiplying larger numbers can be extrapolated from these two examples.

In the multiplication of a two-digit multiplicand and a one-digit multiplier, the item identifier is brailled. On the same line, the multiplicand is brailled, leaving two blank spaces between the end of the identifier and the first digit of the multiplicand. No numeric indicator is used anywhere in the problem.

The paper advance is tapped once. The carriage is positioned in the column containing the units digit of the multiplicand; the one-digit multiplier is written in that location. No multiplication sign is written preceding the multiplier. The paper advance is tapped twice, leaving a blank line under the main portion of the problem. The carriage is positioned so that it is “pointing” to the units column. The student can use his or her finger to easily determine whether the carriage is positioned correctly.

The next step is to multiply the multiplier times the units digit of the multiplicand. The result of that operation is brailled. The backspace key is struck twice to position the carriage correctly for the next step. If the result of the previous multiplication is a two-digit number, the tens portion of that number is held in memory by the student. As suggested earlier, if the student experiences difficulty holding the number in his or her memory, an abacus can be used to temporarily store it.

Once the student has multiplied the multiplier times the tens digit of the multiplicand, if a number were held in the student's memory or if it were stored on an abacus, it is added to the result; the units digit of that number is written. If a tens digit exists in this number, the backspace key is struck twice and that number is written. Once again, if the teacher deems it necessary, the student should advance the paper two lines, move the carriage to the far left and write: ans. = followed by the answer, with a numeric indicator preceding it.

⠼⠢⠸⠲	⠆⠖	#5.	26
	⠶		7
	⠂⠦⠆		182
⠁⠝⠎⠲ ⠨⠅ ⠼⠂⠦⠆		ans. = #182	

The following is a description of the procedure which should be used for a two-digit multiplicand and a two-digit multiplier. The multiplicand is brailled following the problem identifier. The paper is advanced once. The carriage is positioned in the units column. The two-digit multiplier is brailled immediately under the two-digit multiplicand. The paper is advanced two lines, leaving a blank line under the multiplier.

The carriage is positioned in the units column. The multiplication is carried out by multiplying the units digit of the multiplier times the units digit of the multiplicand. The units digit of the answer is brailled. The backspace key is tapped twice. If a tens digit exists in the answer, it is remembered by the student.

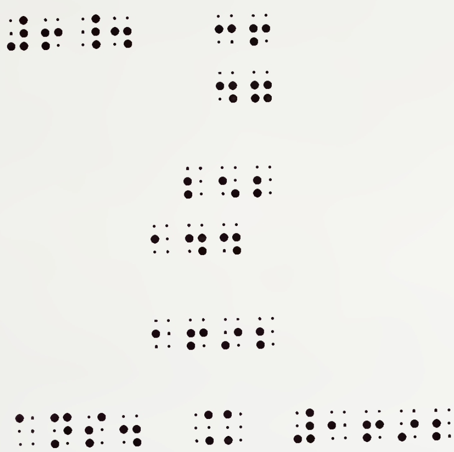
The units digit of the multiplier is multiplied times the tens digit of the multiplicand. If a tens digit exists in the previous multiplication, it is added to the answer and the units digit is written. If a tens digit exists in this result, the backspace key is tapped twice and the tens digit is written. The paper advance key is tapped once. The carriage is positioned in the tens column of the problem.

The tens digit of the multiplier is multiplied times the units digit of the multiplicand. The units digit of the resultant number is brailled. The backspace key is tapped twice. If a tens digit exists, it is remembered.

The tens digit of the multiplier is multiplied times the tens digit of the multiplicand. If a tens digit had existed from the previous multiplication, it is added to the result and the units digit of that result is written.

If a tens digit exists in the resultant number, the backspace key is tapped twice and it is written. The paper advance is tapped twice, leaving a blank line under the partial products. The carriage is positioned in the units column.

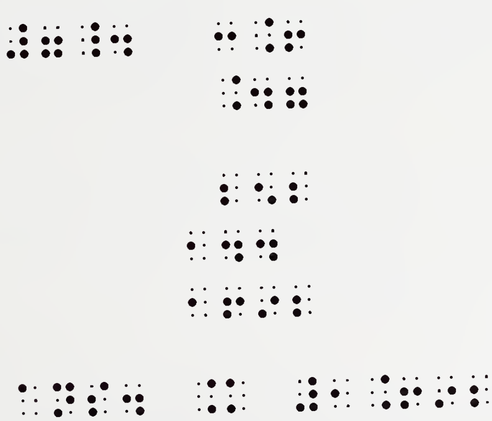
The single digit in the units column is written, and the backspace key is tapped twice. The numbers in the tens column are added, and the units digit of that result is written. The backspace key is tapped twice and the next column is added appropriately. This is done until all columns have been added. Once again, the paper advance key is tapped twice and the carriage is moved to the far left where ans. = followed by the answer, with a numeric indicator preceding it, is written. This procedure can be used with numbers of any size. The number of digits in the multiplier determines the number of partial products.



```
#6. 36
     47

    252
   144

   1692

ans. = #1692
```

Multiplication of numbers with decimals

Multiplication of numbers with decimals follows similar procedures as outlined above. The braille reader must include the decimal point (dots 4-6) in the problem wherever it occurs. In performing the operation, the student ignores the existence of the decimal points and carries out the operation as outlined above. When the final answer has been derived, the position of the decimal point is determined by counting the number of digits which follow the decimal, in both the multiplicand and multiplier. That number of places is "counted over" from the right. The decimal point is then placed in the proper position in the answer.



```
#7. 3.6
     .47

    252
   144
   1692

ans. = #1.692
```

Division of whole numbers

The following is a summary of the recommended format to be followed in writing division problems. An outline of the steps for performing the operation will follow the summary.

On the same line as the problem identifier, the divisor is written. To represent the curved portion of the division symbol, dots 1-3-5 are brailled. Immediately following that symbol, the dividend is brailled. No line of dots 2-5 is brailled above the dividend.

As the operation is carried out, the various digits comprising the answer (quotient) are brailled in a column to the right of the problem. This is done because it is much too difficult and inefficient to braille the quotient above the dividend, as is done in print; that would require moving the paper in and out of the braillewriter, which results in having a portion of the problem disappearing within the braillewriter. It is recommended that the entire problem should be tactually perceivable at all times while the student is performing the operation.

Following are the steps involved in the division of whole numbers without decimals. The answers may contain remainders. Following the recommended format as described above, the divisor and dividend are brailled. The paper advance key is tapped once. The carriage is positioned to the right of the problem. The divisor is divided into the proper portion of the dividend. The one-digit answer is written as the first number in the column to the right of the problem. The carriage is positioned under the units digit of the portion of the dividend which was divided. The number on the right-hand column is multiplied times the units digit of the divisor. That answer is written. If it is a two-digit number or larger, the backspace key is tapped and the tens digit is written. If a hundreds digit is required, the backspace key is tapped twice again, and that digit is written in the proper location.

Once this portion of the operation is complete, the paper is advanced two lines. The carriage is positioned in the units column of this portion of the problem. The proper procedures for subtraction are carried out. Once the result is obtained, the next number in the dividend is “brought down” and brailled immediately to the right of the previous result.

The paper is advanced one line and the carriage is positioned to the far right, under the first number in the column. The division process is repeated. The divisor is divided into the remaining number. The answer is written in the right-hand column.

The carriage is moved to the units column of the previous remainder. The digit which had just been placed in the right-hand column is multiplied times the divisor and the same process is carried out as outlined above. This procedure is repeated until no digits remain “undivided” in the quotient. If a remainder exists, that remainder is written in the right-hand column preceded by the letter “r.”

To “collect” the answer in order to write it in a readable fashion, the student advances the paper two lines. The carriage is moved to the far left where the student writes ans. = followed by a blank space. The numeric indicator is brailled, followed in

the same line by the digits from the right-hand column; these are brailled in sequential order, from top to bottom. If a remainder exists, the student should skip a space and write the letter, “r”, followed with the digits which comprise the remainder.

#8.	32)770	
	64	#2
	130	
	128	4
	2	r2

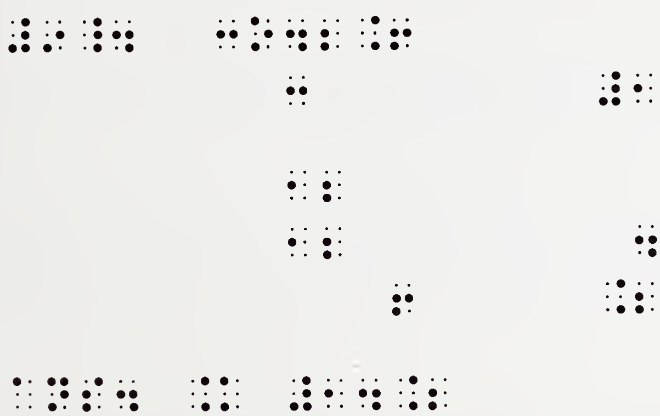
ans. = #24r2

Division of numbers with decimals

The steps involved in the division of numbers containing decimals are similar to those outlined above. Of course, the position of the decimal point in the divisor and dividend and subsequently in the quotient must be accurate. If the divisor does not contain a decimal point, the dividend is written with the decimal point located where it is shown in the original problem.

If the divisor contains a decimal point, then it is moved to the right as many digits as necessary so that it is no longer included in the number. The decimal point in the dividend must be moved to the right as many digits as it was moved in the divisor. The dividend should be written with the decimal point located in the proper space.

The division process is carried out as outlined above. When the divisor is divided into the sequence of digits which contain the decimal point, the one-digit result must be written in the right-hand column preceded by dots 4-6 to indicate the proper location of the decimal point in the answer.



#9.	3)42.6	
	3	#1
	12	
	12	4
	6	.2

ans. = #14.2

If the divisor and the dividend are whole numbers (without decimal points) and the answer is to be found to the nearer tenth, hundredth, thousandth, etc., the following procedures should be carried out. A decimal point should be placed following the last digit of the dividend. The decimal point should be followed by a predetermined number of zeros. The number of zeros is determined by the precision of the answer. That is, if the answer is to be found to the nearer tenth, then two zeros should follow the decimal point. If the answer is to be found to the nearer hundredth, three zeros should follow the decimal point.

```
#10.  17 )110.00
           102      #6

            80
            68      .4

            120
            119     7

ans. = #6.5
```

In this way, the division operation can be carried out to one more decimal position than the level of precision requires, in order to determine if the final decimal point should be rounded up or left as it is written. If the final digit in the column is five or greater, then it should be rounded up. If it is four or less, the decimal digit should be unchanged when the final answer is written.

Fractions

Addition of simple fractions

Following the procedures outlined below, it is possible to use the braillewriter to add, subtract, multiply, and divide simple fractions and mixed numbers. Once again, the strict format for transcribing rules for writing fractions and mixed numbers in braille is changed radically.

Immediately to the right of the problem identifier, the first addend is written. It begins with the opening fraction indicator (dots 1-4-5-6) followed by the numerator. Following that is the simple fraction horizontal bar (dots 3-4). The denominator is

brailled immediately after the horizontal bar. The final portion of the addend is the closing simple fraction indicator (dots 3-4-5-6). The paper advance key is tapped once.

The carriage is positioned immediately under the opening fraction indicator of the first addend. The opening simple fraction indicator for the second addend is brailled in that space (lined up in the same column as the first). The same procedure is then followed for brailling the second addend. The opening fraction indicators should always be brailled in the same braille column.

The numerators and/or the denominators of either the first addend or subsequent addends may have varying numbers of digits. For example, the numerator of the first addend might be a single digit while the numerator of the second addend may be a two-digit numeral. Therefore, the horizontal bars of the addends and the closing fraction indicators cannot be brailled in the same columns without leaving appropriate spaces. Strict transcribing rules require leaving appropriate spaces so that the horizontal bars and closing fraction indicators "line up." However, it is recommended that these spaces not be used in calculation exercises, in order to simplify the process and permit the student to focus on the actual computation.

After the problem has been written, the student needs to determine if lowest common denominators must be found. If the denominators are the same, a simple adding of the numerators should take place. In that case, the paper is advanced twice, leaving a blank line under the final addend. The braillewriter carriage is positioned in the column in which the opening fraction indicators are located. The opening fraction indicator is brailled.

The carriage has now moved to the numerator portion of the answer. The student reads the numerators of the addends and adds them. He or she then brailles that answer, followed by the horizontal bar. The denominator is then written, followed by the closing fraction indicator.

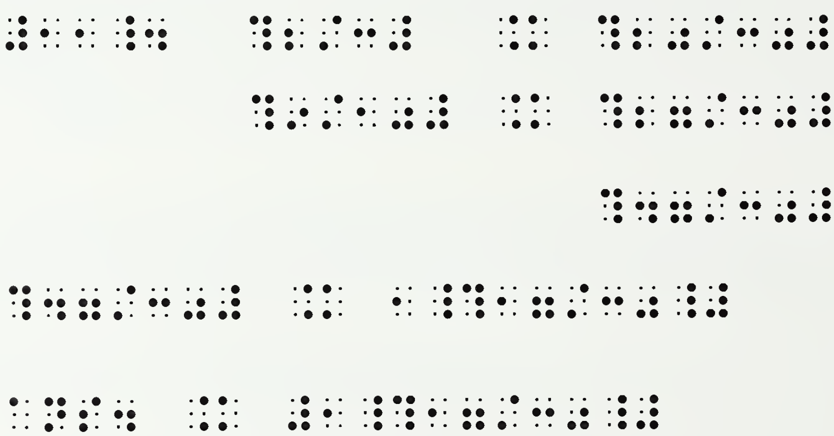
The next step is to reduce the fraction to lowest terms, and then to determine if the numerator is larger than the denominator. If it is, then the fraction must be transformed into a mixed number. This is done by dividing the denominator into the numerator. The result of that division is the whole number portion of the resulting mixed number. The remainder is the numerator of the fraction portion of the mixed number.

The following process should be used to write the various steps in the above procedures in braille. The unaltered answer should be followed by a space and an equals sign (dots 4-6, 1-3). If the answer requires being reduced to lowest terms, an opening fraction indicator should be written. The new numerator is written followed by the horizontal bar. The new denominator is written followed by the closing fraction indicator. If this does not result in a mixed number, the final answer can be indicated

by advancing two lines, moving the carriage to the left, and writing ans. = followed by the fraction.

If the fraction, reduced to lowest terms, can be changed into a mixed number, then the improper fraction (numerator larger than denominator) should be followed by a space and an equals sign. The procedures to change the improper fraction to a mixed number should be used. The resultant mixed number should be written following the equals sign. The whole number of the portion need not be written with a numeric indicator preceding it. It should be followed by the opening mixed number indicator (dots 4-5-6, 1-4-5-6), followed by the numerator.

The horizontal bar is brailled next, followed by the denominator. The final symbol is the closing mixed number indicator (dots 4-5-6, 3-4-5-6). To indicate the final answer, the paper should be advanced two lines. The carriage should be moved to the far left and the abbreviation, ans., should be brailled followed by the equals sign. The equals sign should be followed by a space, and the mixed number is brailled following it. In this instance, the whole number portion of the answer should be preceded by a numeric indicator. The remaining portion of the answer is brailled as indicated above, using the proper mixed number indicators.



#11. 2/3 = 20/30

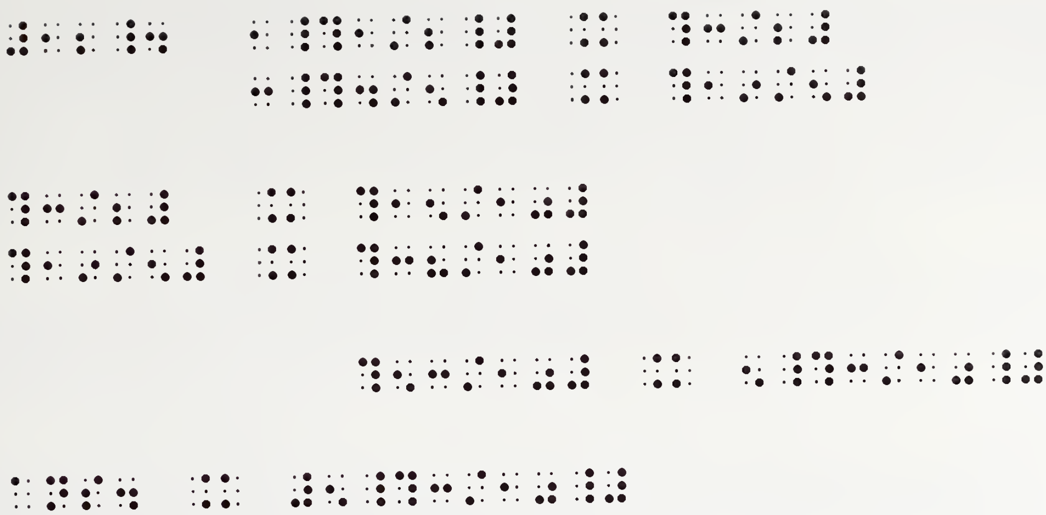
9/10 = 27/30

47/30

47/30 = 1 17/30

ans. = #1 17/30

Addition of mixed numbers follows a similar pattern. The mixed number addends are brailled using mixed number indicators. No numeric indicators should be used. If the fraction portions of the addends require no alteration, the whole numbers can be added, as well as the numerators of the fractional portions. The answer is brailled after the blank line, under the final addend. The correct procedure for adding the fractional portion is carried out. If it requires reduction to lowest terms, the procedures described above should be followed. Lastly, the final answer should be written beginning at the left as described in previous paragraphs. If the fractional portions of the mixed number addends require the finding of common denominators, the procedures for accomplishing that should be followed, as outlined above.



#12. 1 1/2 = 3/2
3 4/5 = 19/5

3/2 = 15/10
19/5 = 38/10

53/10 = 5 3/10

ans. = #5 3/10

Subtraction of fractions

The procedure for subtracting fractions or mixed numbers is very similar to that described for adding fractions or mixed numbers. Of course, the major difference is that the minuend is subtracted from the subtrahend instead of being added.

Multiplication of fractions

The following is a summary of the procedures which should be followed in multiplying simple fractions. The multiplicand is written to the right of the problem identifier. The multiplier is brailled immediately under the multiplicand. A blank line is left under the multiplier. The numbers are multiplied; the result is written as the numerator of the product. The denominators are multiplied; the result is written as the denominator of the product. The product is written in correct Nemeth Code symbols, with an opening fraction indicator, followed by the numerator, followed by the horizontal bar, followed by the denominator, followed by the closing fraction indicator.

If the resultant product requires reducing to lowest terms, the proper procedures are followed. The student should find the largest number which will divide evenly into the numerator and denominator. The result should be brailled following an equals sign. The final answer is brailled under the problem, on the left side of the page.

#13. 3/4
8/9

24/36 = 2/3

ans. = 2/3

If any portion of the multiplication problem is comprised of mixed numbers, those must be converted to improper fractions before the multiplication can take place. Equal signs should be written to the right of the multiplicand and multiplier. The improper fraction equivalents are written to the right of the equals signs. The paper is advanced two lines, leaving a blank line under the multiplier.

The multiplication is then completed. The resultant product is reduced to lowest terms. If it is an improper fraction, it is converted to a mixed number. That final answer is written to the left following the abbreviation, ans., followed by an equals sign.

#14. 2 1/2 = 5/2
4 1/6 = 2 5/6

125/12

125/12 = 10 5/12

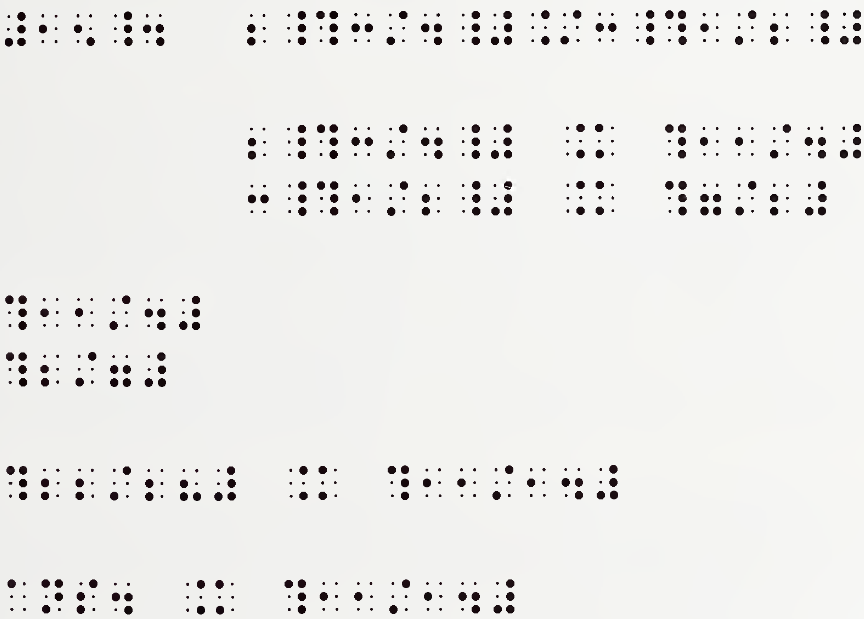
ans. = #10 5/12

Division of fractions

Division of fractions is similar to the multiplication of fractions. The difference in the operation is summarized briefly below. The problem is written as though it were a multiplication problem. The divisor is written under the dividend. When it is brailled, it is inverted. Then, the numerator becomes the denominator and the denominator becomes the numerator. The same procedures are then used as in a multiplication problem. Reduction to lowest terms, and/or conversion to a mixed number is done if necessary.

A word of explanation is called for here. Division of fractions is represented in print in horizontal form. The first fraction along the horizontal line is the dividend and the second number following the division sign is the divisor. Attention must be paid to inverting the divisor only in order to carry out the operation correctly.

If one or both of the portions of the division problem is or are mixed numbers, these must be converted to improper fractions before the operation is carried out. Once the conversion has taken place, the same procedures are used as those which are described above.



#15. 2 3/4 ÷ 3 1/2

2 3/4 = 11/4
3 1/2 = 7/2

11/4
2/7

22/28 = 11/14

ans. = 11/14

Use of the braillewriter as a calculation tool is essential for a blind youngster's fundamental understanding of the steps involved in the four basic operations of addition, subtraction, multiplication, and division of both whole numbers and fractions. Blind students who rely solely upon the talking calculator have no opportunity to learn these steps. The abacus is an excellent tool for teaching the steps involved in arithmetic operations, but it does not afford the student the opportunity to emulate how sighted individuals perform these operations. A point of emphasis: the braillewriter should not be the only tool available for blind students to perform arithmetic calculations. As a student becomes more proficient with the braillewriter, greater emphasis should be placed on the use of the abacus. As the mathematical concepts become more complex, and after the student has mastered the basic operations, greater reliance can be placed on the talking calculator.

The Abacus

The abacus, in particular the Cranmer abacus, is certainly one of the most effective calculation tools for blind children—for both low and high achievers—when used either alone or in conjunction with other devices. It allows concrete manipulation, leading to a more meaningful understanding of numbers than does the use of calculators, and it provides an alternative to lengthy and involved calculations done on the braillewriter, although the ability to work with these two tools is also very important and discussed elsewhere in this manual. In fact, the combined use of the braillewriter and the abacus can be very effective; students can use the abacus to check their work on the braillewriter and, if there is a discrepancy, rework the problem using both tools. Generally, it is recommended that the student progress from using the more cumbersome tool to the less cumbersome tool. For some students, picturing the working of problems on the abacus has even increased their ability to carry out calculations mentally.

The abacus is also useful because of its speed, accuracy, portability, and flexibility. It can be used for educational purposes to support a good foundation in addition, subtraction, multiplication, and division. It can also be used to carry out calculations involving fractions and decimals, as well as an aid in completing arithmetic operations included in higher level mathematics. It can also be used for independent living skills such as recording phone numbers, or tabulating costs while shopping.

Developing skill in the use of the abacus

Skill in the use of the abacus depends on several factors. One of these is readiness. For example, in order to begin working on the abacus, children must understand basic number concepts, be able to count, and know the partners or complements that make up the numbers up to ten. These concepts should be taught concretely with manipulatives, by forming and rearranging sets of objects. Students also need to learn that some beads on the abacus stand for one, some for five, some

for ten, and so on, as well as knowing the basic concept of place value. As students work with sets, they can set relevant numbers on the abacus. As they learn about place value, they can reinforce this concept with the abacus. As they become comfortable with all the partners of the numbers up to ten, they can set simple number statements on the abacus. Familiarity with the abacus should start early. Young students should be encouraged to use it in limited ways as they develop number concepts; they can use it as a calculating tool later, as their skills progress.

Students also need the manipulative skills required to operate the abacus itself. Some teachers prefer starting young children on the enlarged abacus because of the larger beads and the greater space between the beads; students can then make the transition to the smaller abacus when appropriate.

A third requirement for the student's success with the abacus is his or her teacher's competency and attitude. Teachers must take responsibility for developing their own skill to the level sufficient to teach their students the skills that will benefit them. Teachers must also convey a positive attitude about the use of the abacus, making its use—and the effort required to learn its use—rewarding for the student.

Teaching approaches

There are several approaches to teaching the use of the abacus. Since one method might not work effectively for all students, teachers should be familiar with several methods. The most commonly used approaches are:

- the partners or logic approach,
- the secrets approach,
- the counting method, and
- adaptations or combinations of these approaches.

Each is briefly described below, with an example (3+4=7) worked out according to that approach.

The logic method or partner method focuses on understanding the "what" and "why" of the steps in solving a problem on the abacus. It requires that the student know the partners or compliments of the numbers up to ten (5=2+3, 5=1+4). Verbalizing the steps and the reasons for each movement made on the abacus is an important feature of this approach. At first, the teacher must explain the steps and reasons as the student works through the problem. Then the student should verbalize the process as he or she works the problem. Over time, this "conversation" can be shortened, and finally the process is internalized. This approach would benefit students who can follow the explanation and can understand, and even enjoy, the logical concepts involved.

Example: "The problem is 3+4. What number comes first? The answer is 3. So we set the 3 on the abacus. Now we need to add 4. Do we have 4 more ones to add? No. Since we don't have enough ones to add, we can add the 5 bead (set 5). But 5 is too many; we only wanted to add 4. So we'll have to clear the extra bead or beads. What is 4's partner in 5? The answer is 1. So we'll clear one extra bead. Now what is our answer? The answer is 7."

Example: "The problem is 3+4. What number comes first? The answer is 3. Set 3. Can you set 4 directly? No. What is the smallest amount that can be set that is greater than 4? The answer is 5. Set the 5 bead. How many more is 5 than 4? The answer is 1. Clear 1 bead. What is the answer? The answer is 7."

The secrets method focuses on the process of moving the abacus beads in a particular sequence, following a specific set of rules for different numbers and operations. It does not emphasize the understanding of that process, rather the rote memory of the bead movements. It would be appropriate for students who would benefit from a manipulative process they could rely on, without having to fully understand the principles behind each step of that process.

Example: "The problem is 3+4. What number comes first? The answer is 3. So set 3 (raise three earth counters). Now we want to add 4. In order to do that, we must set 5 (bring down a heaven counter) and clear 1 (clear one earth counter). What is our answer? The answer is 7."

The counting method has the student count each bead as it is added or subtracted, moving from the unit beads to the 5 beads (but counting only 1 for all beads). There are also specific rules regarding certain numbers and operations, but fewer than the full set of secrets. It does not emphasize understanding the concepts behind the bead movements. This approach could also be appropriate for youngsters who would benefit from a manipulative process they could rely on, without having to understand each individual step.

Example: "The problem is 3+4. What number comes first? The answer is 3. So we set 3. Now we want to add 4. To do that, we push up another unit bead (count 1), then another unit bead (count 2), then push down the bead above the counting bar (count 3), and clear all four beads under the counting bar. Finally, push up one more unit bead (count 4). What is the answer? The answer is 7."

There are several resources available which demonstrate how to teach the use of the abacus employing the above approaches: *Abacus Made Easy* (Davidow, 1975) utilizes the logic approach. *The Japanese Abacus—Its Use and Theory* (Kojima,

1954), and *Detailed Instruction on the Use of the Cranmer Abacus* (Foster, 1974) utilize the secrets approach, although each in a different manner. *Abacus Basic Competency* (Millaway, 1994) utilizes the counting approach. *Use of the Cranmer Abacus* (Livingston, 1997) utilizes both logic and counting approaches.

Teachers of blind children have made a variety of modifications to all of these approaches in order to meet the individual learning styles of their students. For example, students included in the regular classroom for much of the time can work their addition and subtraction problems from right to left to coincide with the way the teacher works through the problem with the class.

An example of a more specific modification relates to division, and is sometimes referred to as the "subtraction method" of division. The divisor is placed to the far left on the abacus, then 2 columns are left blank, followed by the dividend. The quotient is the sum of partial answers obtained as the student works through the problem, and is placed to the far right.

Example: 3075 divided by 15:

```
15 into 3075 = 200
       - 3000
15 into   75 =   5
         - 75
                205
```

Another specific example of a modification of the logic method involves multiplication of one, two or three digit multipliers and one or two digit multiplicands. For example, in the problem 93x25, the first factor (93) is set in the billions place, the second factor (25) in the millions place, and the answer in the thousands and hundreds places. Instead of working from the outside in, the entire multiplicand is multiplied by the first digit of the multiplier; then the entire multiplicand is multiplied by the second digit of the multiplier.

Example: 93x25

```
starting at the left, multiply 90x20 = 1800
next, multiply 90x5                  =  450 (clear the 9 tens)
next, multiply 3x20                  =   60, so add 60 (clear the 2 tens)
next, multiply 3x5                   = __15, so add 15 (clear the 3 and 5)
                                       2325
```

Strategies for teaching use of the abacus

In addition to modifying general approaches to teaching the abacus, teachers have found several strategies that can help to facilitate students' learning of this skill. Some of these are included here:

- Familiarity with the abacus should be started at an early age as the child begins working on number concepts; in the elementary grades it can be used to support the learning and understanding of operations and calculations, as well as the use of fractions and decimals. It can easily be used in conjunction with the braillewriter; by middle school, students should be proficient at using the abacus, making less writing with the braillewriter necessary. Guidelines for combining the use of these two tools are presented in another section of this manual.
- It is important that the student develop a positive attitude about using his or her abacus. One teacher told her student that he won it, making him feel special and eager to use it. Providing simple but relevant tasks in which the student uses the abacus would also be helpful in motivating the student.
- The abacus can be used for a variety of functional and motivating classroom activities, such as keeping scores for games, tabulating scores on daily quizzes, using in simple money games (1=pennies, 5=nickels, 10=dimes).
- Games such as an abacus “bee” can add to the fun of learning the abacus. Teams can be selected in a variety of ways. Individuals can compete against individuals in a predetermined order that could rotate. Scores or winning teams can be determined in a variety of ways. Each student should have his or her own abacus in this or similar games.

Since blind students cannot see the gestalt of where their beads are placed on the abacus, it is extremely important to teach them place-keeping habits. This will be especially critical when they are dealing with problems involving multiplication, division, decimals, fractions, and any problems involving zeros.

Two abaci—placed either one above the other or side by side joined by a coupler—can be used effectively to record and sum partial products or answers on one while working additional steps of the problem on the other.

The abacus can be used in conjunction with Chisenbop or fingermath. Teachers have begun instruction with either of these skills and found that transition to the other was relatively smooth, due to the similarity of the terminology (set/press, clear, etc.) and to the relative values of certain beads/fingers representing 1, and certain beads/thumb representing 5.

Secrets Chart

Some addition and subtraction can be carried out “directly” on the Cranmer abacus, that is, appropriate beads can be moved without having to make any exchanges. For example, 3+1, or 5+2 or 8-5 can be done directly because moving the beads with the real value is sufficient to solve the problem. However, this is not always

the case. In a problem such as 3+2, 3 beads can be set, but 2 more beads cannot be set directly. Instead, the five bead must be set and the four previously set beads must be cleared. The logic method would have the student verbalize this process. The secret method would offer steps for moving beads in a specific manner to arrive at the right answer. A table of these secrets for addition and subtraction is presented below.

To Add	**Secrets**
1	Set 5, clear 4
1	Clear 9, set 1 left
2	Set 5, clear 3
2	Clear 8, set 1 left
3	Set 5, clear 2
3	Clear 7, set 1 left
4	Set 5, clear 1
4	Clear 6, set 1 left
5	Clear 5, set 1 left
6	Set 1, clear 5, set 1 left
6	Clear 4, set 1 left
7	Set 2, clear 5, set 1 left
7	Clear 3, set 1 left
8	Set 3, clear 5, set 1 left
8	Clear 2, set 1 left
9	Set 4, clear 5, set 1 left
9	Clear 1, set 1 left

To Subtract	**Secrets**
1	Clear 1 left, set 9
1	Set 4, clear 5
2	Clear 1 left, set 8
2	Set 3, clear 5
3	Clear 1 left, set 7
3	Set 2, clear 5
4	Clear 1 left, set 6
4	Set 1, clear 5
5	Clear 1 left, set 5
6	Clear 1 left, set 4
6	Clear 1 left, set 5, clear 1
7	Clear 1 left, set 3
7	Clear 1 left, set 5, clear 2
8	Clear 1 left, set 2
8	Clear 1 left, set 5, clear 3
9	Clear 1 left, set 1
9	Clear 1 left, set 5, clear 4

Fingermath

Fingermath or Chisenbop is an alternative manipulative approach to calculations which allows students to be actively involved in carrying out those calculations. In some case, teachers have introduced fingermath first and transitioned students into the use of the abacus; in other cases, teachers have started students on the abacus and introduced fingermath as a supplemental technique. In either case, students do not seem to find the transition from one to the other difficult (Maddux, Cates & Sowell, 1984).

While the abacus has many advantages, for some students it also presents several problems. The beads may be difficult to manipulate for children with motor weakness or limited coordination. The abacus also requires many hours of training and practice in order to become proficient. While fingermath also requires training and practice, it may be simpler to learn for some children, the finger movements are simpler, and no equipment is needed. A disadvantage of fingermath is the difficulty in writing the answer, since it must be remembered before moving the hands to record it, especially if two hands are needed for recording.

Fingermath has numerous similarities to the abacus. For example, the values of the fingers on the right hand are the same as the values of the beads in the ones column on the abacus, with the four fingers corresponding to the unit beads and the thumb corresponding to the five bead. The values of the fingers on the left hand are the same as the beads in the tens column on the abacus with the same correspondence of fingers and thumb to the unit and five beads. The operational language and the use of "exchanges" or "secrets" is also similar. For example, in the problem 4+1, the student using fingermath would first "press" (set) the four fingers on the right hand all at once; then, to add one, he or she would simultaneously press the thumb while lifting the four fingers. The resulting answer is five (only the thumb remains pressed). Similarly, the student using the abacus would first set four unit beads; then, to add one, he or she would set a five bead and clear the four unit beads. The resulting answer is five (only the five bead remains set).

When teaching students the techniques of fingermath, the teacher should emphasize:

- the student's correct identification of the numbers which he or she is pressing, since he or she cannot visually check the answer;
- a sequence of simple pressing (setting) of numbers, followed by counting, simple addition and subtraction which does not require exchanges or secrets, problems involving such exchanges, continuing calculations (e.g., 3+1+1+4+10-5), and later, multiplication and division if appropriate;

- practical problem-solving to illustrate the convenience and application of fingermath skills (e.g., problems involving game scores, money, etc.);
- use of the technique in conjunction with other calculation methods and aids; and
- eventual internalizing of the hand movements enabling the student to calculate by mentally imagining those movements.

Mental Math

The ability to calculate mentally with efficiency is a very important skill for all students, but especially for visually impaired and blind students. Using the braillewriter, and the abacus can be very labor intensive and time consuming, and calculators have their own limitations (see the discussion on calculators elsewhere in this manual). The more efficiently students can estimate, calculate, and check the reasonableness of answers in their heads, the more facile they will be at using numbers, in both schoolwork and independent living skills. These strategies should be taught to students as soon as they begin to count and work with simple numbers.

In order to manipulate numbers and calculate mentally, students must understand the concept of "complements" or "partners" of numbers. For example, in addition and subtraction, the student needs to know that the number 5 is made up of addends of 2 and 3, or 1 and 4 (complements, partners). Likewise, the number 12 is made up of 3 and 9, or 6 and 6, or 10 and 2. In multiplication and division, the student must know that the number 24 is made up of factors of 2 and 12, or 6 and 4, or 8 and 3.

Teaching approaches

While there are many individual techniques for estimating and calculating mentally, most strategies involve one of the following four basic approaches:

a) *decomposing numbers* — breaking apart numbers into meaningful and useful units or groups that can be easily recomposed

Example:	54+23	37-13
	54+20+3	(37-10)-3
	74+3	27-3
	=77	=24

b) *making easier numbers to work with*—putting numbers together that are easier to use, often by changing the order of numbers

Example:	3+86+7
	(3+7)+86
	10+86
	=96

c) *substituting numbers* — replacing values with equal values that are easier to manipulate

Example: 1/8x720
720 dvided by 8 (substituting a whole number for a fraction)
=90

Example: .75x32
3/4x32 (substituting a fraction for a decimal)
(32 divided by 4)x3
8x3
=24

d) *compensating* — rearranging numbers so they are easier to work with, either by changing a number and then adjusting the answer, or by adjusting both numbers so there is no need to change the answers

Example: Adjusting the answer
23+29
23+(30-1)
53-1
=52

Example: Adjusting the numbers
23+29
22+30 (Adjusted numbers — lower the 23 by 1, raise the 29 by 1)
=52

Strategies for developing mental math skills

Following are several examples of strategies which may help students develop skills in counting and using the basic operations.

Addition

- Using the idea of complements, the student can adjust numbers to make adding a lot easier.

Example:	63+30	673+99	27+49
	60+30+3	673+100-1	27+50-1
	90+3	773-1	77-1
	=93	=772	=76

- Students can handle larger, more complicated numbers by starting their addition by adding the largest place values first, then next largest, etc.

Example: 3849+4563
3849+4000=7849
7849+ 500=8349
8349+ 60=8409
8409+ 3=8412

- Students could also simplify their addition by adding the tens or hundreds together first, and then adding the units.

  Example: 59+76
  50+70=120 (five tens and seven tens are twelve tens)
  9+ 6= 15 (adding the units)
  =135 (15 added to 120 equals 135)

  (The student could write down subtotals of 120, 15, 135 as needed as he or she calculated in his or her head.)

- Students who have difficulty remembering many of the facts, can use the additive principle, doubles (2+2, 3+3) or facts or "partners" for numbers up to 10 (7+3, 6+4), and derive other facts from these (4+3=1 less than 4+4).
- For the addition of nines, the student can keep in mind that the one's digit in the sum is always one less than the number added to the nine. For example,

$$\begin{array}{r} 9 \\ \underline{+\ 5} \\ 14 \end{array} \qquad \begin{array}{r} 9 \\ \underline{+3} \\ 12 \end{array} \qquad \begin{array}{r} 9 \\ \underline{+9} \\ 18 \end{array}$$

Subtraction

- The ability to understand the concept of partners or complements comes in handy in mental subtraction, as with other operations.
  1. Start by practicing subtracting partners from numbers up to 10.

     Example: 8-4=4 7-4=3 5-3=2 9-4=5

     10-8=2 10-5=5 10-7=3 10-6=4

  2. Continue the process by subtracting partners from numbers 20, 30, 40, etc.
     Example: 20-8=12 20-7=13

     30 -__=8 30 -__=17 40-22=__

  3. Continue practicing subtracting two digit numbers from 100, and then numbers larger than 100.

Example: 111-77 Think: 23 is the complement of 77 in 100 (+11).
23+11=34

- Another approach involves subtracting numbers from smaller units which are closer to the actual subtrahend, and then adding the remaining portion. Always start by subtracting digits from the same number of digits immediately above it, then deal with the remaining amounts.

Example: 100-43 (think of 43 from the next largest unit, or 50)
50-43=7
7+the remaining 50=57

Example: 100-67
70-67=3
3+the remaining 30=33

Example: 400-326
330-326=4
4+the remaining 70=74

- When subtracting a number from a number that is a power of 10, use the complements that make up the numbers 9 and 10.

Example:
$$\begin{array}{r l} 100 & \\ \underline{-\ 56} & \text{(the complement of 6 in 10 is 4,} \\ 44 & \text{the complement of 5 in 9 is 4)} \end{array}$$

- Students can also "balance" numbers by adding the "same difference" to both to make them easier to work with.

Example:
$$\begin{array}{l l r} 25\ (+1) & & 26 \\ \underline{-\ 9}\ (+1) & & \underline{-\ 10} \\ 16 & & 16 \end{array}$$

- "Balancing" can also be done with decimals.

Example:
$$\begin{array}{r l r} 2.64\ (+.04) & & 2.68 \\ -\underline{\ 1.96}\ (+.04) & & -\underline{\ 2.00} \\ 4.68 & & 4.68 \end{array}$$

Multiplication

- Students can learn to think in patterns or arrays by using a "thinking model" with naturally occuring arrangements like those occuring in egg cartons, pop bottle cases, buttons on cards, cookies or candies packaged in rows,

etc; then children can develop their own arrays. This approach can also be used with auditory cues; for example, how many times do you hear 2 taps, 3 rings, etc.?

- Emphasize the associative properties of the factors in multiplication. For example, remind the student that 3 fours is the same as 4 threes, 2 sixes is the same as 6 twos (rotate an egg carton 90 degrees to illustrate).
- Multiplication is repeated addition—if the child knows that 2 fours is 8, then 3 fours is 8 plus another 4, or 12.
- Use the concept of doubles—if the child knows that 2 sixes are twelve, then 4 sixes is twice as much, or 24.
- When multiplying by multiples of 10, students can just remember to add zeros—add one zero for each time that a number is multiplied by a multiple of 10.
- As with addition, students can also think of numbers as quantities rather than digits, and start by multiplying the largest units first.

Example: 27x138 (think of 138 as 100 and 30 and 8, etc.)
20x100 equals 2000
20x30 equals 600, added to 2000 equals 2600
20x8 equals 160, added to 2600 equals 2760
7x100 equals 700, added to 2760 equals 3460
7x30 equals 210, added to 3460 equals 3670
7x8 equals 56, added to 3670 equals 3726

- Another way of using this "front end" multiplication follows:

Example: 214x6
200x6=1200
10x6= 60 (added to 1200=260)
4x6= 24 (added to 1260=1284)

- When multiplying mentally by 5, 50, or 500, the student can simply multiply by 10, 100, or 1000 and then divide by 2.

Example: 5x268
2680 (268 multiplied by 10)
2680 divided by 2
=1340

Example: 5x188
1880 (188 multiplied by 10)
1880 divided by 2
=990

- When multiplying mentally by 9, 99 or 999, students can multiply by 10, 100, or 1000 and then subtract one multiplier.

  Example: 37x99=37x(100-1)
  =3700-37(37x-1) (think 700-37, use partners)
  =3663

- Another application of this "rounding" and adjusting approach could be used for many other numbers.

  Example: 8x498=8x(500-2)
  4000-16(8x-2)
  =3984

- When multiplying decimals by .1, .01 or .001, students can keep in mind that the number of decimal places is equal to the total number of decimal places in the factors.

  Example: .24x.001 = .00024
  .001x.008 = .000008

- Whenever multiplying decimals by tens, the decimal point "slides" over one place to the right; when multiplying decimals by hundreds, the decimal point slides over two places to the right, etc. In the same manner, whenever dividing decimals by tens, the decimal point slides over one place to the left; when dividing decimals by hundreds, the decimal point slides over two places to the left, etc.
- A game called "Buzz" (Petreshene, 1985) can help students practice reviewing the products of a given number times many different multipliers. Specify a number, and have the student start counting from 1 to 100. Whenever he or she comes to a multiple of the specified number, the student says "buzz" instead of the actual number. For example, if 4 were the designated number, the student would count "1, 2, 3, buzz, 5, 6, 7, buzz, 9, 10, 11, buzz" and so on. Extra challenge could be added by calling any number which contains the designated number, "busy": "1, 2, 3, busy-buzz, 5, 6, 7, buzz, 9, 10, 11, buzz, 13, busy, 15, buzz, 17, 18, 19, buzz, 21, 22, 23, busy-buzz" and so on. Variations might include time restraints, or brailling the numbers and words.

Division

- Again, the student can use his or her understanding of partners or complements—how numbers are made up of other numbers—to calculate mentally.

Example:	6 divided by 3=2	35 divided by 5=7
	66 divided by 3=22	70 divided by 35=2
	390 divided by 130=3	490 divided by 70=7

- Just as it is sometimes easier to calculate multiplication mentally by dealing with the larger units within the factors, and then adding the progressive products, it can also help to use a similar approach when dividing larger numbers.

Example: 3726÷27

27 into 3700 goes	100 times,	leaving 1026
27 into 1026 goes	30 times,	leaving 216
27 into 216 goes	8 times	
	138	

The student can keep a running record of his or her intermediate quotients and add them progressively (e.g., 100+30+8, or 138)

Counting and general ideas

- Practice mentally calculating number chains; these can be used for any operation, or they can be combined to work on a variety of basic facts.

Example: 5+3-4+3-5
6+4-2x3÷3

- Practice with familiarity in the sequence of numbers.

Example: What comes after?
10, __ 23, __ 199, ___ 268, ___

Example: Skip counting ahead
2,4,6,_ 3,6,9,_ 5,10,15,__ 15, 30, 45, __

Example: What comes before?
__, 16 __, 25 __, 200

Example: Skip counting backwards
10, 8, 6, _ 15, 12, 9, _ 350, 300, 250, ___

- Play games such as "secret number" where the student draws a card with a numerical relationship on it and another student must provide the answer to that relationship.

Example: 2 less than 87 11 more than 40 7 below 81

- Play a form of Jeopardy where the student provides the answer and other students provide different descriptions of that number.

  Example: What is 2? (factor of 4, square root of 4, remainder of 10-8)

  Example: What is 100? (product of 10x10, half of 200, 1 more than 99)

The reader is referred to the the Resources and Reference sections of this manual for several books which provide numerous additional ideas for developing mental math skills at different grade levels.

Talking Calculators

During initial instruction in arithmetic operations, the braillewriter and abacus should be the major tools used in calculations. The talking calculator should only be used as a reinforcer for skills learned with the braillewriter and abacus until a student masters the fundamental concepts involved in computation. As the student becomes more proficient with the braillewriter and abacus, and demonstrates understanding of basic mathematics concepts, progressively less emphasis should be placed on the braillewriter and more emphasis should be placed on the use of the talking calculator.

Eventually, the calculator is likely to become the student's major tool for performing calculations. This is especially recommended in the advanced study of mathematics, for example, algebra, where the emphasis is upon learning content far advanced from the simple performance of arithmetic calculations. Steps should be taken to give the blind student the most efficient tool to use so he or she is not expending inordinate amounts of time in the performance of arithmetic calculations, but rather devoting study time to mastering the subject matter content of a course.

While the calculator is the most efficient method for a blind student to perform arithmetic calculations, it has two major disadvantages. First of all, reliance on the talking calculator does not afford the student the advantage of practice in the underlying steps needed to perform the calculation. One can use a calculator without actually understanding the underlying mathematical principles. Secondly, heavy reliance upon use of the calculator results in the loss of instant recall of the basic addition, subtraction, multiplication, and division facts. If a calculator is used too early, while the youngster is learning the basic facts, he or she will not have immediate recall of mathematics facts in his or her repertoire of skills. On the other hand, the calculator enables students to solve problems which are challenging and interesting. Since intellectual development is often at a higher level than that of arithmetic skill (Baggett, 1995), enriching mathematics lessons to include some problems which are at the student's level of thinking while beyond his or her arithmetic skill level will encourage curiosity and persistence in mathematics. This can be achieved with the use of calculators. *Breaking Away from the Mathematics Book* (Baggett, 1995) is an excellent

resource for ideas about teaching the functions of a calculator, as well as calculator based games and activities.

Talking calculators may be used in educational settings in a variety of ways, including:

- To practice basic facts
- To improve the speed and accuracy of computational skills
- To provide students who have mastered basic operations with a competitive calculation tool
- To provide an alternative to the abacus
- For advanced mathematics and science calculations that are too complex for computation with the braillewriter
- For students with additional motor disabilities which impede the use of the abacus, or with cognitive disabilities which hinder comprehension of mathematical concepts or rote learning
- For independent checking of computation, whether using mental math or other aids such as the abacus
- For assisting in other school subjects such as bookkeeping, business, geography, and cooking.

Although a variety of calculators with distinct characteristics are available to aid in a number of unique tasks, the following characteristics are essential:

- Large separated keys for increased accuracy
- Adjustable volume of auditory output
- Adjustable speed of auditory output
- Number keys set apart from the "off" key to avoid errors.

There are several additional specific questions to consider when selecting the most appropriate calculator for an individual student. These include:

- What type of output is needed?
- What are the applications for this calculator?
- What specific functions or type of calculator is needed (square root, logarithms, standard deviation, adding lists, etc.)?
- What is the range of numbers to be used? If more than eight, it may be necessary to use a calculator with scientific notation.
- What is the level of accuracy (number of decimal places) needed?

- Is portability a factor? If it needs to be portable, it should have rechargeable nickel-cadmium batteries.
- Is there a need for a review function (to review a calculation already entered) or a read key (to reread the number on the visual display)?

Teaching strategies

- If at all possible, students should learn to understand the concepts of operations first, with manipulatives, hands-on problem-solving, and aids such as the abacus and braillewriter, before relying on the calculator to provide the answers. During these early stages, the calculator can serve as a motivating reinforcer and checking aid for their work.
- It is important to stress development of the ability to estimate and consider the reasonableness of the answers they obtain.
- When considering hand use, the type of task may be relevant. When working from auditory dictation, the dominant hand could be used to calculate, while the other can locate and hold the place. When working from problems in a book, the dominant hand can be used to read the book or to keep place in the book, and the other hand can calculate. Students should develop an effective strategy for themselves.
- The student must devise an efficient finger placement approach. The first three fingers might be placed across the top row of digit keys, moving from this position to depress other keys, and returning to this position before moving on to other keys. Some calculators have a tactile marker on a central key (often the 5 key), and for some students, this base position may be very helpful.
- Several characteristics of calculators can be confusing to younger children. For example, calculators with auditory output often read the decimal point automatically; simply explain this or avoid this type of calculator with younger children. Several calculators read the division sign as “over”; again, simple explanation and comparison to writing methods should avoid confusion.

Electronic notetaking devices

The Braille ‘N Speak and Braille Lite, manufactured by Blazie Engineering, are very small, portable computers which combine speech synthesis and braille; the Braille Lite also has an electronic braille display. The most current versions of these devices contain a calculator function which can be used to perform a variety of different mathematical operations. These devices can be used by blind students of any age, as tools for performing mathematics calculations.

Using these devices, whole numbers and decimals can be added, subtracted, multiplied, and divided; square root and percentages can also be calculated; and the precision can be easily set to 12. Strings of calculations can be performed at one time. The results can be stored in 26 different memory locations, and can be retrieved later for inclusion in other strings of calculations. A random number generator is included.

The calculator function also includes a scientific calculator in which trigonometric and logarithmic calculations can be performed. In addition, it contains translation tables in which values in the English system and metric system can be converted to the other system (e.g., kilometers to miles and miles to kilometers). The conversion tables contain the following:

Centigrade to Fahrenheit; Fahrenheit to Centigrade
centimeter to inches; inches to centimeter
kilometers to miles; miles to kilometers
liters to gallons; gallons to liters
kilograms to pounds; pounds to kilograms
grams to ounces; ounces to grams

Activities for teaching calculator skills

- Practice entering numbers, storing and retrieving from memory, and basic operations as well as calculator functions by having students enter numbers which are meaningful to them (i.e., their age, number of siblings, telephone number, etc.). Have one child call out a number, while others enter it in their calculator.
- Provide the student with an actual braille menu from a popular fast food establishment. Calculate the total bill for a variety of selections (i.e., sandwich, salad, beverage, and dessert); teach the % function and add tax to the total as an additional challenge.
- Braille a grocery or other retail store receipt in its entirety. Help the student to identify different parts of the receipt, such as the date, the store number, register number, clerk or cashier number, and item prices. Have the student check the total price; this will also serve as a check on keystroke accuracy.
- Use the calculator to compare unit prices. Have the student find out the prices on 4 different sized packages or brands of the same product, e.g., different sized cans of peas. In addition to walking through the steps involved in making a decision about which would be the most prudent purchase for an individual consumer (taking serving size, perishability, and so forth, into account), the student can practice multiple operations and the use of the memory function.

- In a group of three or more students, calculate the individual cost of sharing a pizza equally. Although a cardboard cutout would work, real pizza would be the most effective prop! Use calculators to divide the total cost of the pizza by the number of pieces; and to divide the total number of pieces by the number of students. Again, use calculators to figure the cost for each individual. An additional challenge would be to have an odd number of pieces; the students have to figure out how to distribute the cost.

TACTILE DISPLAYS AND GRAPHICS

The use and complexity of graphics has increased dramatically in education. Textbooks in science and mathematics are now predominantly graphics with supporting text. Job opportunities in science, mathematics, and computers, as well as other areas, will also require the ability to deal with increasing amounts of graphical infomation. However, even with the advanced technology that greatly increases the accessibility of information to blind persons, the area of graphics continues to pose a major problem to this accessibility. If blind students are not taught the skills and provided the appropriate materials to study subjects that contain a considerable amount of graphics, they will be denied the opportunity to fully participate in education and employment.

Tactile displays can include three dimensional aids—either real items or models—or two dimensional aids. Tactile graphics usually refer to two dimensional aids, and are made of points, lines and shapes. Such tactile graphics can be developed by using a collage method or a tooled method. The collage method involves adding a variety of materials to the front of a board or sheet of paper. These can then be used in their master form or they can be copied via a heat/melting/swelling process. Tactile graphics can also be developed by using tools such as tracing wheels, leather working tools, or braille compasses. Teachers might select either method depending on the particular concept to be addressed. Often teachers might also choose to combine tactile displays and tactile graphics of either or both types, to introduce and reinforce concepts, or to teach a concept and then provide a way for the student to work related problems.

When dealing with tactile displays and graphics, it is very important to remember that these materials are meant to be felt and not seen. It is also very important to remember that visual perception and tactile perception are not the same; information is not received in the same manner by the visual and tactual senses, and that information is not processed by the same areas of the brain. For example, a three dimensional drawing of a house cannot be rendered into a tactually perceivable drawing which makes sense to a blind person. The three dimensional drawing represents perspective which cannot be perceived tactually. Warren (1984) addresses this discrepancy with the following statement:

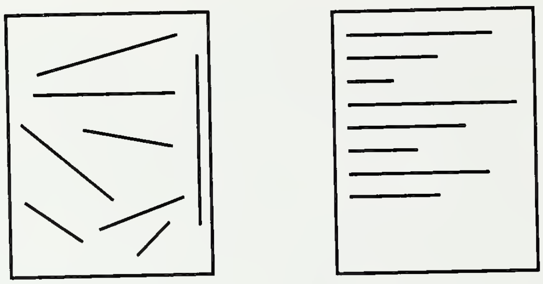
> Touch for the blind child does not serve the same function that sight does for the sighted child. Touch may provide information about the same or similar events in the environment (such as spatial relations) that vision does, but touch differs in major ways from vision, particularly in its successive manner of delivering information and in its far less detailed discrimination of the spatial field (p. 144).

Therefore, images that relay information clearly visually do not necessarily relay information clearly tactually. This is particularly true of many pictures and drawings utilizing perspective; these cannot be accurately perceived by blind readers.

Guidelines for Designing Tactile Displays

There are a variety of situations which call for the information in graphic displays or tactile graphics to be maintained as is, or which suggest that the information be modified somewhat in order to make the tactile display usable by the blind reader.

In some cases, the size of the information must remain the same (e.g. if measuring is to be done), but the layout can be changed to facilitate locating and measuring lines or areas.



In some cases, the size or scale may be changed to facilitate the reader's perception of important details and space/line differences (NOTE: at least 1/4 inch space between entities is necessary for the tactile reader to perceive them as separate; more space is needed if there is a difference in the height of the entities). If the need to enlarge results in having to split the diagram into more than one part, these splits should be made in such a way that it does not interfere with the concept being presented. Furthermore, the parts should be presented in a careful sequence, with special attention provided to help the student tie them together. A diagram containing the gestalt of the display (but with less detail) should be presented either as an introduction to the separate parts, or as a summary, whichever approach works best for the student. The student might also consider the use of a marking system to help keep track of the part he or she is working on with respect to its location on the "big picture" diagram.

The size of the symbols used must also be considered. The absolute size of the symbol is important because a) the blind reader cannot look closer to increase the size of the image on the retina, as the visual reader can, and b) the blind reader cannot use magnifiers to increase the symbol size, as the visual reader can. Furthermore, the relative size of the symbol makes a difference in the tactile reader's ability to recognize it efficiently. It is more difficult and takes more time to recognize shapes if they are of a different size than the one originally presented. This suggests avoiding varying the

size of symbols on displays or keys, unless the difference in size denotes different meaning. Many students may also benefit from working on size constancy.

The relative position of shapes or symbols is also an important consideration. The same shape, presented in different positions, is not readily recognized as being the same by most blind readers. Therefore, the position of symbols or pictures should not be changed if it is important that the student recognize these symbols as being the same. In addition, as with size constancy, teaching shape constancy may be helpful. Of course, sometimes the change in a symbol's position denotes different information, and is therefore necessary. Careful orientation to any display and its descriptive information and key is always very important.

Sometimes the layout, size and even format of the display is changed. This is acceptable and, in some cases, necessary, in order to present the original concept or purpose of the display in a way discernible to the tactile reader. The blind child cannot perceive the whole layout at once, and has great difficulty skipping over unimportant details while searching for essential information when a cluttered format is presented. Therefore, tactile displays must be carefully analyzed before rendering, and presented in an orderly and uncluttered format so the reader can locate all the significant information, not be confused by unimportant information, carry out problem-solving tasks (e.g. measuring, comparing), and keep track of his or her progress while working.

Often visual displays contain patterned or shaded backgrounds; in many cases, these background patterns are not relevant to the purpose of the diagram. To the tactile reader, rendering these background patterns with various textures can be extremely distracting and confusing. Textured areas slow down the speed and decrease the accuracy of reading tactile diagrams, and make tracking lines through these areas very difficult. Therefore, avoid rendering such backgrounds unless they are an important part of the information to be presented.

It is also often necessary to use simpler shapes to represent more complicated shapes or pictures for easier readability. This is appropriate as long as it is not necessary to maintain the same picture as is used in the original display. For example, The Boehm Tactile Analog presents the same concepts but substitutes simpler symbols for the pictures used in the original tasks. Many primary mathematics books use fairly complicated pictures (animals, birds, tractors, etc.) to indicate members of sets to be counted; the numbers of items in the sets are important for counting or comparison, but the exact duplication of complicated pictures is not important, and is often confusing.

Careful decisions should be made with regard to what elements of information to include and how to best present them through the use of tactile displays. When designing and producing tactile diagrams, the following additional suggestions may be useful:

- Focus on the purpose of the tactile graphic; this will help to identify what information is critical to include.
- Edit the printed display carefully before making it into a tactile display, deciding on important information to include, unnecessary elements which may be confusing, and ways for clearly rendering areas and symbols.
- Consider the individual student who will be using the tactile aid; his or her background, experience with graphic materials, cognitive and language level, etc. will help to insure that the particular aid is appropriate.
- Consider the life of the aid—will it need to survive for a long time, be used very frequently, under a variety of conditions? Choose materials accordingly.
- Proofread the tactile diagram (both the master and the copy) with your fingers, not just your eyes.
- Identify critical features necessary to make the graphic aid accurate (e.g. intersections on graphs, extension of number lines to cover problems to be solved, exact areas and fractional parts if they are to be measured, etc.).
- When designing tactile graphics, keep non-essential information to a minimum—it can be confusing to the reader; avoid background textures unless they convey important information. It is especially difficult for blind readers to disregard unimportant details or background textures; therefore, grid lines could be left out if they are only background material and have no relationship to the diagram itself or to the student's ability to solve the problem involved.
- Background lines such as grids or guidelines can be helpful, even necessary, if precise location of points must be found in order to interpret the graph; in such cases, these gridlines or guidelines should be rendered with lower key types of lines than those used for more important information such as data point lines, etc.; similarly, axis lines should be clearly presented, but they should not override the purpose of the graph.
- Scale is very important if it is involved in the work to be carried out or the information to be obtained; it is not important if it does not play a role in the problem-solving at hand, or provide useful information, or if it causes the user to become confused by crowding or blurring important symbols or lines. It may be much more effective to enlarge details which are critical to the use of the diagram so that they can be perceived more accurately. If scale is modified, this should be described on the diagram or on a facing page with the key or other relevant information.
- The size of the map or diagram itself is important; generally, the area to be examined should be the size of one or two hands, depending on the concept and the student. If diagrams are too small, they may be too crowded and confusing; if they are too large, it may be difficult to experience different

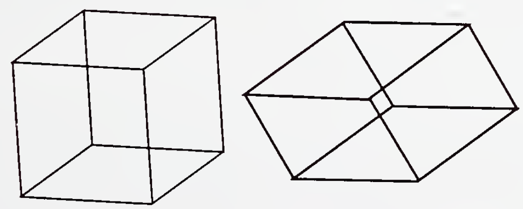
areas as being connected or related. Unless accurate scale is necessary, small diagrams with much detail might better be rendered using an enlarged version with an emphasis on critical features. Complicated diagrams could be split into several sections (an appropriate size for one- or two-handed reading) with a simplified overview diagram and descriptive note, as long as the use of the diagram would not require repeated flipping back and forth among parts in order to solve a problem.

- Material which may not fit efficiently on a vertical page might fit well on a horizontal page; some materials, such as graphs, might be more easily perceived if rendered on a horizontal axis. If a display is repositioned, caution should be taken to insure that the information itself remains accurate (e.g. curved lines curving downward in one position may end up curving upward in another position). If the display's position is changed, this should be described in notes provided with the key and other relevant information.
- If information is presented in a random order in the visual display, this same information would be better presented in some organized fashion in the graphic display so as to avoid the potential for missing some of the entries.
- A wide enough margin should remain around the edges of the actual display so that the diagram can be thermoformed if necessary, or so that it can be bound or hole-punched for a notebook (if a diagram is to be bound or placed in a notebook, it should face away from the binding so that the binding will not be in the way of the reader's hands).
- Spacing between lines and symbols is critical in order for the user to perceive symbols and lines as distinct features. For many students, even experienced users of tactile graphic aids, spacing of 1/4 inch between entities (symbols, lines) is necessary to distinguish separate symbols and lines.
- Spacing between braille characters and other lines or symbols is also critical and should also follow the 1/4 inch rule; additional space may be necessary if braille characters or symbols are placed next to information which is presented in especially high relief.
- Shapes can be rendered in higher relief, and lines at lower elevations to be more easily discriminated by the reader.
- Important lines should attract the reader's attention immediately; this can be accomplished by using sharper or highly contrasted textured lines or by raising the elevation of these lines.
- Symbols and angles should be of sufficient size to be accurately perceived by the tactile reader.
- Lines which require measuring should be of sufficient relief that the reader can easily use a ruler or protractor; if the perimeter of an area is to be measured, place the labels of the line segments inside the shape so as not to interfere with the use of the ruler.

- When rendering bar graphs, keep bars close enough so the length relationships can be easily read.
- When small differences must be detected on a display, use a combination of length or size with relative texture (e.g. longer bar has coarser texture); this may help increase reading accuracy.
- When making line graphs, use different types of lines to denote different information, rather than using the same type of line with different labeling.
- When rendering diagrams to coincide with classroom materials, symbols may need to resemble those in the print display for clarity of classroom discussion; however, this often results in using symbols which are tactually very confusing. Clear explanations and descriptive information along with simpler symbols may be more appropriate.
- All symbols should be defined and explained. Descriptions or keys to accompany a tactile diagram could be placed on the same page, but out of the way of the diagram itself, or on a separate facing page. Such placement can help to reduce clutter on the diagram. This is especially important if key information is in braille, since its size cannot be reduced to fit in smaller spaces within the chart or diagram, as is possible with print.
- While it is generally a good idea to use abbreviations when labeling diagrams in order to reduce clutter, it is better to use two-cell braille labels than one-cell braille labels since they enhance orientation and are less easily confused. Grade two braille should be used in labeling as long as the student is proficient with it.
- A braille label should not break the integrity of a shape; it should be placed either inside or outside the shape, depending on the task involved and the space available.
- Capitalization need not be followed in braille labeling, unless the capitalization is a critical element of the information (e.g., a diagram involving line AB and line ab).
- It might be helpful to have a list of the Nemeth Code and other relevant symbols frequently used in mathematics available to the students for quick reference as they work with their diagrams.
- Tactile diagrams should be marked to indicate the top of the diagram for more efficient orientation.
- It is extremely difficult to effectively represent 3 dimensional objects in 2 dimensions, even with the use of raised line drawings using dotted lines as opposed to solid lines to denote those edges and angles which would be "unseen" if viewing the actual item visually. Raised line drawings often look very clear to the sighted individual because the concept has been experienced or because perspective drawings are more easily understood visu-

ally. The assumption that these diagrams are equally effective for blind users is incorrect.

- A "collapsible cube" can be made to demonstrate both 3 dimensional and 2 dimensional frames: 2 rigid square planes are joined by 4 flexible vertical edges ("arises"). The cube can be used upright (3D), or the bottom frame can be held firmly on a flat surface with the top frame held directly above it, then the top frame is moved at a 45 degree angle (or any angle) to the side until it rests flatly on the desk surface. By moving the frame from a 3D to a 2D representation of the cube, the student may develop a better understanding of the derivation of a 2 dimensional representation of a 3 dimensional cube. Additional examples and illustrations are provided in Neumann's "Demonstrating the Relationship Between Three-Dimensional Figures and Their Two-Dimensional Representations to Blind Students of Mathematics", included in the Reference section in this manual.



- Numbers, letters and words written in braille on separate paper, cut out and glued to the diagram often result in distracting "boxes", especially if thermoform copies are used; if possible, plan ahead and braille necessary sections first before producing the graphic elements of the diagram.
- Use lead lines only when absolutely necessary; when including them, however, use very low key lead lines that will not interfere with the important lines of the diagram. Try to place the beginning and end of lead lines as close as possible to the features which they connect, without interfering with the connected labels or symbols.
- When possible, use consistent tactile symbols for frequently occurring visual symbols, pictures or concepts; exceptions occur when different symbols representing the same concept are actually part of the material to be taught.
- Since many texts use the same basic diagrams repeatedly, with only the labels on the diagrams changing, construct a sturdy set of basic braille diagrams (e.g., on foil sheets) and make a set of braille stick-on labels which can easily be added and removed as needed.

Excellent examples of a wide variety of tactile diagrams and graphs, demonstrating the above listed tips and guidelines, as well as equipment and materials, are available from *Tactile Graphics* by Polly Edman (APH), the *Tactile Graphics Kit Guidebook* (APH), *Tangible Graphics Teacher's Guide* (APH), and *Guidelines for Mathematical Diagrams* and its supplement (Braille Authority of North America), and

John Gardner's *Tactile Graphics, An Overview and Resource Guide.* These and others are included in the Resources section of this manual.

Teaching Students to Use Tactile Displays

In order to be successful at handling and interpreting a variety of models and tactile graphics, blind students must first have many opportunities to experience real-life concepts, handle real objects, then models, and finally two dimensional and symbolic representations with guidance from their teachers. Dr. Snorre Ostad of Norway stresses the importance of this latter stage in his study, *Mathematics Through the Fingertips* (1986), by reminding us that mathematics processes and symbols are abstractions. Students' abilities to understand abstract (pictorial) representations of real items need to be developed before working with the abstract processes involved in mathematics. Snorre stresses the need for early, repeated exposure to, and manipulation of, tactile images of concrete objects not only as a pre-reading strategy, but also as a pre-mathematics strategy.

Developing well-designed tactile displays is not enough to insure that blind students will be able to interpret and use them effectively. Successful reading of tactile displays involves not only the legibility of the display, but also a) the students' strategies for exploring and interpreting the tactile graphics, and b) the students' knowledge of spatial and geographic concepts. Yet numerous studies have shown that blind students have poor haptic skills, especially related to tactile discrimination, spatial orientation, systematic searching, and tracking and tracing.

Furthermore, blind children often do not "see" the whole or the gestalt at once, as sighted viewers do, and so they must experience many concepts sequentially, part by part. This synthesizing can be much more difficult and time-consuming, and requires that students examine objects, displays, and graphics carefully and systematically in order to a) insure that they experience the entire field and do not miss any important information, and b) take advantage of relationships that will help in synthesizing or putting together the whole picture.

Students also need specific experiences and training in the concepts and skills needed for reading and interpreting tactile information, including:

- the discrimination of real objects, shapes and two dimensional symbols and lines;
- tracking and comparing these lines, shapes and symbols; and
- developing and maintaining spatial orientation as they explore and work with models and tactile graphs.

Furthermore, since it is extremely difficult for blind students to recognize different-sized shapes as being the same, and even more difficult for them to recognize

shapes which are rotated in space as being the same, practice in these perceptual skills is also very important, as is the ability to determine when changes in size and spatial orientation should be interpreted as meaning the same thing or when they indicate different information.

The appropriateness and importance of teaching students to be more proficient in reading tactile materials is highlighted in Bentzen's (1982) statement:

> The skills of display users improve with instruction and practice. Therefore, appropriate materials and curricula should be developed for the instruction of blind students in the use of tangible graphic displays. Teachers should be taught to teach their use (p. 402).

In addition to the above suggestions, the following guidelines may help to facilitate your student's success in working with a variety of tactile displays and graphics:

- Be sure that students have a good understanding of basic concepts before working with tactile graphics, so that they can concentrate on the new concepts and other information presented in the diagrams.
- Whenever possible, provide the student with experience with the real thing before expecting him or her to develop concepts from representations.
- Use 3 dimensional objects or models alongside symbolic displays whenever possible, but especially when introducing your student to a new concept or type of graphic display (e.g., cans for cylinders, pointed drinking cups for cones).
- Use real life objects and the environment to convey positional/geometric concepts (e.g., the table is parallel to the floor, the corner of the room or desk is a right angle).
- Multisensory displays may be more motivating and provide more conceptual information; however, it is important not to bombard the student with too much varied information at once. Attention can be guided from one part of a concept to another.
- Tactile displays that are interactive (e.g., involving the rearrangement movable parts) may also be more motivating and provide additional information about a concept; again, however, care must be taken to guide the student through the purpose and handling of the interactive elements.
- Provide careful sequencing when moving from 3 dimensional (concrete) to 2 dimensional (symbolic) presentations.
- To assist the student in understanding the relationship between object and tactile representation, have him or her put simple items on paper and make thermoform copies, and then compare the item and the paper display.

- Use a logical sequence of simple to complex displays (e.g. diagrams with single lines before double lines, simple symbols before complex symbols, simple concepts before more difficult concepts, etc).
- When selecting models to represent actual items or concepts, insure that the critical features of that object or concept are present in the model to be used (e.g. wheels that move on a car, animals that are not hollow and open and that have feet that are separate and not attached to a platform or "ground"). Furthermore, when introducing your student to models or tactile graphics, emphasize the critical features and relationships first; if additional information is appropriate, it can be addressed secondarily or in succeeding displays.
- Teach students to develop a systematic search pattern for exploring tactile displays. Research suggests that this skill is one of the most critical for success in handling tactile graphics. This should include using one or both hands to scan the entire field to get a gestalt for the type of information presented, the layout and format of the information, or the location of a key or other descriptive information. While students may use vertical, horizontal or circular search patterns, a vertical approach is often most efficient because it involves fewer sweeps to cover more area more quickly with less overlapping. Using a systematic search pattern will also insure that the entire field was examined and will prevent the student from missing important information.
- Work with students on discriminating types of lines (horizontal, vertical, diagonal, parallel, intersecting, angles, shorter/longer comparisons), areas, spaces, and symbols (matching, comparing, same/different, noting directional differences).
- Work with students on tracking lines (single, double, intersecting, embedded in areas).
- Teach students to orient themselves to the display efficiently, including critical cues (e.g. title, key, scale notes). Have students read the key or other descriptive information before looking for details or attempting to solve problems; this will help to focus attention on "key" points and make important information more immediately recognizable.
- Create a simple graph using raised line graph paper (APH) and place it on a small bulletin board that can be used at the student's desk; the child can record data points with pushpins. If the graph needs to be preserved for future reference, remove the pins and squeeze a dot of puff paint over each pinhole.
- For younger children, two strategies for comparing distance or space between symbols are a) using the width of the finger to "measure" distances

between symbols, and b) estimating the relative ratio of one distance to another (e.g. “about half as long”, or “only a little longer”).

- Teach students and their classroom teachers how to produce their own graphic displays.
- Provide a short list of Nemeth Code symbols in both braille and print to accompany tactile graphs; this could be helpful to both the student and the classroom teacher.
- Familiarize students with the types of graphics used on tests such as the Stanford Achievement Test so that they will not be penalized by a lack of ability to read tactile diagrams.
- Students can practice and apply their developing skills with graphs by tracking the percentage of correct answers in a variety of subjects.

Examples of teaching aids

- Paper folding, or origami, can be a fun and motivating technique for teaching a variety of concepts. It actively involves students in discovery and helps their understanding of basic constructions. It also supports the learning of such concepts as fractions, angles, line segments, area, altitude, and more complex geometric concepts. Some things to keep in mind when using this technique:

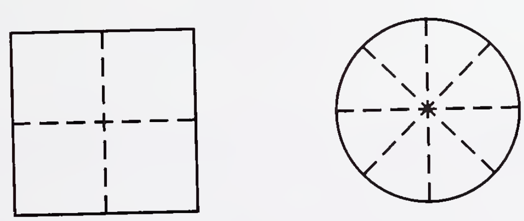
    Use paper with enough body to make a good crease but not so heavy as to make it unmanageable; braille paper is a good choice, but brown wrapping paper is also effective and is less expensive, more flexible and offers greater possibilities regarding the size of the shapes.

    Prefold creases for students with motor impairments.

    Prescore for students who might be able to use the technique independently.

    Have an abundance of pre-cut shapes readily available, since students may use many in a short time.

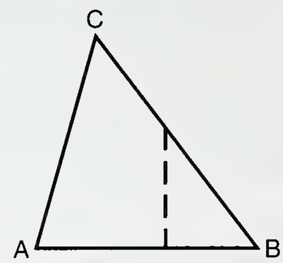
Example 1:



Note: folding instructions on following page

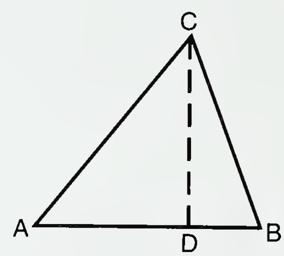
(Fold in half, 2 halves = 1 whole)
(Fold in half, then in half again, 4 quarters = 1 whole)
Continue process

Example 2: Perpendicular line segments



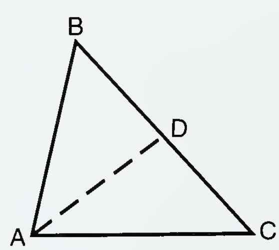
(Fold point B onto line AB to form a line perpendicular to AB.)

Example 3: Altitude of triangle

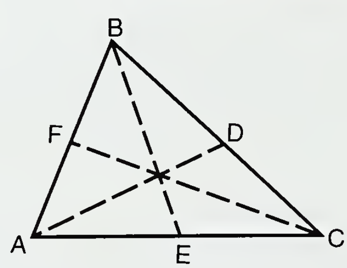


(Fold line AB so that the line passes through angle C;
this line D is the altitude of triangle ABC.)

Example 4: Bisector in center



Example 5: Incenter



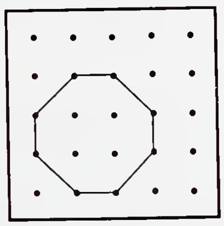
(Bring line AB down to line BC and fold, forming line AD or the bisector of angle A; repeat the procedure to form bisectors for angles B and C.)

(Discover the in center X, or the point where all 3 bisectors intersect.)

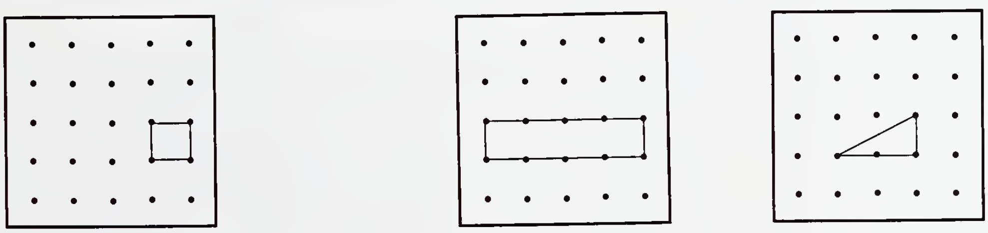
Additional examples of using origami in geometry are presented in Tinsley's "The Use of Origami in the Mathematics Education of Visually Impaired Students" (included in the Reference section in this manual).

- Geoboards are boards of wood or masonite with nails placed at regular intervals much like a pegboard; some have line segments like grids displayed while others do not. Line segments, shapes or other designs can be made by arranging rubber bands or string or other appropriate materials. Numerous games and activities can be carried out on a geoboard to teach such concepts as shapes, positional concepts in two dimensions, patterns, area, perimeter, and fractions.

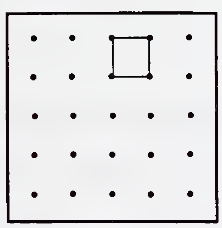
  Example 1: Reproduce this design, shape



  Example 2: If it takes 1 paper square to cover this area, how much paper does it take to cover these areas?



  Example 3: What is the length around the square (1x4)?



Teachers can find many suggestions for teaching concepts with geoboards from activity books in teacher stores, such as those at different grade levels by Learning Resources; additional ideas are included in Marion Walter's *Use of Geoboards to Teach Mathematics*, and Patricia Brosnan's *Visual Mathematics Using Geoboards*, both included in the References section in this manual.

Materials Used to Develop Tactile Displays and Graphics

Teachers can use an unlimited variety of materials and tools to produce the tactile graphics that their students will use. Creativity, problem-solving skills, preparation time available, equipment available, the particular type of display needed, and its anticipated use will all influence the specific choices teachers will make at any given time. The materials and tools listed below reflect some of those commonly used by teachers to produce tactile graphics, but are only a small sampling of what is available.

For tooled lines:

brailler or slate and stylus for lines (dots 1,4; 2,5; 3,6)

dry ball point pen with foil or plastic sheets

braille compass

braille protractor

standard size adapted protractor notched every 5 degrees and a standard compass: since height variations in the lines and points are critical, some points are made high enough for the student to "lock" the zero of the protractor onto while others are open in the middle to anchor the tip of a compass (APH)

stitching techniques (e.g., stitch a line with heavy cord)

screwdriver and hammer for dashed line

nutpick

Sewell Raised Line Drawing Board with plastic sheets (APH)

tracing wheel with a soft underlying surface such as a neoprene pad or rubber mat (the type of mat determines height of line: e.g., rubber produces a high line; cardboard, a low (faint) line)

carbon paper face up to transfer the pattern or design to the back of the page

carbon paper face down to transfer the pattern or design onto the front of the page

For lines applied to the top of the diagram:

screenboard (APH) for drawing outlines or shapes that are tactually perceivable

Wikki-Stix (bendable, wax covered string)

yarn, heavy thread, or cording and glue

covered florist wire

carpet warp thread

Form-a-line or Chartpak graphic tape (the narrower tape can be curved to form circular figures)

models made with flexible straws taped together

TacPaint (a program developed to modify MacPaint or SuperPaint to create raised line drawings that are tactually readable; available from Marie Knowlton at the University of Minnesota)

puff paint

dry spaghetti

pipe cleaners

stitching techniques

raised line graph paper (APH)

heavy construction paper glued in place for bar graphs

pantograph

ric-rac for wavy lines

microcapsule paper or flexipaper (expands, when heated, wherever black lines have been drawn); provides highly discriminable lines; additional lines can be added repeatedly, increasing detail on a diagram

Tactile Image Enhancer (Repro-tronics, Inc.)

Swell-Form Graphics Machine and Swell-Touch Paper (American Thermoform Corp.)

Graph-it (graphing software from Blazie Engineering)

Braille Lite (electronic refreshable braille display from Blazie Engineering)

For making points:

Swail dot inverter

beads

snaps

paper punch: sheets of cork with adhesive backing

Tactile Graphics Kit (APH)

freehand drawing stylus (tongs)

Materials and resources for general graphing and diagramming purposes:

corkboards, chipboards, rubber mats, neoprene pads, and push pins

glue gun

toothpicks for applying glue

fabric paint

styrofoam with braille graph paper, push-pins, rubber bands

heavy duty aluminum foil

variety of tapes

variety of fabrics or textiles
braille calendars
tactile representations of braille clockfaces
magnet board, graph paper, magnetic strips and shapes
paper for cutting and folding
tactile histogram boards and peelable dots (SAVI)
Graphic Aid for Mathematics (APH): cork composition board with a rubber mat which has been embossed with a grid of 1/2 inch squares; thumbtacks, push pins, rubber bands and/or string can be used to indicate axes and shapes, to construct plane figures, number lines, or graph systems
felt boards with Velcro shapes
geoboards (APH) for use with pushpins or nails, and yarn, string, or rubber bands
leather working tools
shape templates
Hi-marks (Kentucky Industries for the Blind, Louisville)
Swail dot inverter
braille labeler
stereocopiers for producing plastic copies of raised line diagrams (Thermoform, Matsumoto, and Repro-Tronics)
Pixcells Mathematics (software from Raised Dot Computing)
Tactile Graphics Kit (APH) embossing tools and teacher's guide
Tactile embossing kits (Gilligan Tactiles and National Braille Press): plastic sheets with tools and templates
Tangible Graphs (APH): lessons for developing skills in effective interpretation of graphic displays
Guidelines for Mathematical Diagrams (BANA): examples and suggestions for creating raised line drawings
Introduction to Map Study I and II (APH): sequential lessons for progressing from real objects to representational layouts and symbols
Chang Mobility Kit (APH)
NOMAD (APH): an electronic, touch sensitive pad with accompanying software; can be programmed to provide auditory feedback regarding a tactile display; requires connection to a computer
Geometric and Volume Aid (GAVA) (APH): set of small cubes which join together to make larger cubes and other shapes
Box of Blocks: Geometric Forms (APH): set of 3 dimensional forms of a variety of shapes
Mitchell Wire Forms (APH)

COLLABORATIVE AND INCLUSIVE STRATEGIES

The Transdisciplinary Model

Since the inception of IDEA, which mandates placement in the least restrictive environment, inclusion (or the regular education initiative) has become the predominant model in special education placement. Effective educational programming in an inclusive setting requires intensive and ongoing collaboration of all members of a student's educational team. There is movement away from the traditional multi-disciplinary model, where experts work independently (and usually on a "pull-out" basis) on assessment, development of separate goals, and instruction related to their particular area of expertise, meeting occasionally to report progress to the team.

Advantages of the team approach

Research indicates that the transdisciplinary teaming model, with integrated special services occurring during the regular program in the regular classroom, is the most effective way of delivering instruction. In this model, everyone works collaboratively on the same goals, sharing responsibility for assessment, planning, sharing of information, problem-solving, and decision-making. Experts in each area are responsible for reporting and monitoring progress in goals most related to their area of specialization, as well as "role release", or training of other team members in the best practices of their specialized area as they apply to an individual student.

Inclusion in the regular classroom provides a continuity of curriculum for the student, with fewer interruptions in the day. The student can readily compare his or her skill level and achievement to that of his or her peers. Instruction in compensatory skills, such as the use of adaptations or assistive technology, is more effective and generalized when taught within the regular curriculum, in regular classroom activities. In addition, when a student attends his or her neighborhood school, he or she is more likely to be a part of the greater community, participating in community activities where functional application of skills will take place.

When a student is enrolled in mainstream classes, both special and general educators have increased opportunities to observe academic and social progress in order to make valid comparisons with peers, ensuring that the student's development is age- and ability-appropriate. It is the ideal way to determine if the student is using time and assistive devices appropriately, and whether he or she is using effective notetaking strategies. There is the opportunity for teachers to combine expertise in order to teach more effectively.

Inclusion eliminates the problem of quality (or perceived quality) of mathematics instruction from a special education teacher, and reduces the issues created when a mathematics teacher is not versed in accommodations and adaptations for blind students. At the elementary level, inclusion also eliminates the inconvenience, to the classroom teacher, of having to send the student out of the classroom for mathematics instruction at important times of the day. This is particularly important when the schedule changes and the student misses a different class activity in order to receive mathematics instruction.

Implementation of the team approach

Implementation of the transdisciplinary teaming model requires training in collaboration and team teaching techniques. Regularly scheduled, frequent (at least weekly) time for planning and reviewing progress, airing problems, and discussing different approaches and instructional strategies is essential. There is a need to teach strategically, providing support groups or individualized additional instruction for students who need more time to practice facts, or who could benefit from enrichment or an extension of the curriculum. Meeting individual needs for methods or materials based on their particular learning style and/or strengths will help students build successful experiences in mathematics, and improve their confidence in mathematics-related skills.

In the ideal teaming situation, teachers share in planning, presenting lessons, and checking assignments. It is vital that students (as well as teachers!) see both classroom and special educators as teachers, rather than one as a teacher, the other as a helper. Professionals share personal and professional strengths, and appreciation for each other's expertise. Both teachers assume responsibility for instruction and for all students, including sharing success and frustrations, planning, evaluating, and problem solving. The teachers move back and forth between direct and indirect support. This system helps improve instruction by working collaboratively with strengths, joint efforts to solve problems, generation of creative methods, reduction of professional isolation, increased understanding of roles of different professionals, and a reduction of the stigma of special education (Pugach & Johnson, 1995).

Mathematics teachers are the specialists in mathematics, particularly in middle school and high school. Once a student masters basic concepts, including Nemeth Code, then the mathematics teacher should teach mathematics, while the teacher of visually impaired students is responsible for teaching any new code information, and transcribing materials into braille, raised line drawings, and tactile graphics. Each uses his or her particular expertise, working closely together to facilitate learning. In high school, while co-planning and teaching may be impractical, it is vital that the special educator facilitate an ongoing system for communication with the mathematics specialist. This may take the form of memos, telephone contact, regularly scheduled

tutoring sessions, or perhaps assistance from the teacher of visually impaired students in the administration and evaluation of tests.

Strategies for team teaching

Cooperation between grade level classroom teachers and special education personnel, including paraprofessionals, is necessary for inclusion in mathematics to succeed. Small homogenous groups for instruction in at least part of a lesson enable the teachers to adjust content according to ability levels while implementing modifications or adaptations. A small group also enhances student involvement and immediate feedback from the teacher. Large, heterogeneous groups are usually effective for introducing a new concept or skill. Grouping decisions should always be viewed as temporary (dynamic, or contingent, or flexible grouping), depending on the nature of the lesson and individual needs of the students.

There are a variety of effective methods for co-teaching. Different strategies should be used for different circumstances, depending on which would be most effective for a particular lesson. Some of the possibilities include:

- Teachers each take half the class and teach a concept or skill to mastery.
- One teacher provides guided practice, for the entire class, in a concept previously taught but not mastered, while the second teacher moves around the room providing individual assistance and monitoring individual performance.
- The teacher of visually impaired students works with an individual student, using adaptive techniques and materials during the regular lesson.
- One teacher demonstrates alternative teaching techniques for the other teacher.
- The teacher of visually impaired students provides related enrichment lessons or units.
- The teacher of visually impaired students assumes total responsibility for a sub-group within the class.
- The teacher of visually impaired students provides instruction for all students in the use of peer tutors and partner learning techniques.
- The teacher of visually impaired students serves as tutor for groups of children having difficulty with a particular concept or skill.
- The teacher of visually impaired students can be responsible for a mathematics learning center in a primary level classroom.
- Both teachers assume responsibility for modification of the curriculum or assignments, if necessary, including quantity, simplification of format or instructions, or assessment procedures.

- Both teachers are involved in the development of the IEP.
- Both teachers participate in conferences with parents.

Activities for teaching in an inclusive setting

- Institute a mathematics "problem of the day," developed and graded by the teacher of visually impaired students. This will help both teachers to track the progress and skills of individual students relative to the rest of the class.
- To encourage the development of skills in self-advocacy, the teacher of students with visual impairments can periodically develop a poorly-planned lesson, present it to the student, and role-play potential strategies for soliciting appropriate information or assistance.
- In secondary level mathematics classes, teaching assistants can be trained in a protocol for spoken mathematics (see Appendix B for an example), and to make "on-the-spot" tactile diagrams.
- In some large school districts, a high school mathematics teacher is assigned to a resource room for students with visual impairments for one period each day. The mathematics teacher provides individualized instruction in any of the mathematics courses in which the resource students are enrolled, while the teacher of visually impaired students provides adaptations.
- Enlist the assistance of the Orientation and Mobility Specialist to reinforce mathematics concepts in "real life" situations.

SPOKEN MATHEMATICS

Following is the text of a presentation from Abraham Nemeth, Ph.D. to members of the Research and Development and Science and Engineering Committees of NFB, on August 5, 1995:

> When I was studying mathematics at the college and post graduate levels, I used sighted readers for accessing text materials, since braille materials in mathematics were almost entirely unavailable. Even as a college professor, particularly during my early years, I continued to use sighted readers, volunteers, paid, and teaching assistants, largely for the same reason.
>
> No standard protocol exists for articulating mathematical expressions as it does for articulating the words of an English sentence. Therefore, it was necessary for my reader and me to come to some agreement as to the most efficient method, for I needed to turn in assignments as a student; and since I often prepared handouts for my students and for faculty seminars as a professor, it soon became clear that the same protocol should be used when I was dictating and my reader was doing the writing. Little by little, a surprisingly simple protocol evolved. I can teach it to a new reader in about 15 minutes. The most important feature of this protocol is the principle that anything that was read earlier or anything that will be read later should never affect what I write now in response to my reader's current utterance. I appropriated this principle from the Nemeth Code which I had already developed and which was, of course, the code that I was using for the writing of mathematical text.
>
> At our recent convention in Chicago, this matter of "MathSpeak" came up at our Science and Engineering meeting. At the conclusion of this item, John Miller, our chairman, gently but firmly instructed me to prepare a write-up of this method and post it on the Internet. What follows is my attempt to comply.
>
> The speech generated by this protocol is not exactly what a professor in class would use, but it is absolutely unambiguous and results in a perfect Nemeth Code transcription. It avoids largely unsuccessful attempts by a reader to describe the notation he sees, accompanied by the shouting and gesturing that such attempts at description engender. When each of a number of readers abides by this protocol, it is a snap to record the information, and makes much better use of the time spent with a reader.

As Raised Dot Computing has amply demonstrated, Nemeth Code can be converted into correctly formatted print notation. And this print notation can be converted into speech by using the protocol described in this paper. Thus, speech to Nemeth Code to print to speech demonstrates that these three systems are notationally equivalent. The ability to convert from one form to another is a distinct benefit to a blind person engaged in the field of mathematics, whether as a student, a teacher, or a worker in a professional field, and gives him a competitive edge.

These considerations strongly suggest that serious thought be given to refining "MathSpeak" and making it a standard by which complex mathematical notation can be communicated to a blind person in verbal form. It could also serve as the basis for transmitting mathematical notation electronically when ASCII is not capable of conveying the notation (many mathematics symbols have no ASCII representation) or when ASCII codes which represent notation are likely to be unfamiliar to the recipient.

Following are the symbols which Nemeth believes are the essential ones to be standardized. Some differ—more abbreviated and therefore requiring less time to speak—from the forms recommended by Lawrence Chang, also a blind mathematician. Dr. Chang, in fact, recommends altering his as desired, as long as there is consistency for an individual using several readers. His *Handbook for Spoken Mathematics (Larry's Speakeasy)*, 1983, is very comprehensive and addresses not only reading to blind students, but also speech synthesis issues. His publication is reproduced in Appendix B.

"MathSpeak" (Abraham Nemeth)

Lowercase letters:

pronounced at face value without modification

never combined to form words, particularly trigonometric and other function abbreviations

examples: s i n, not sine; t a n, not tan or tangent; l o g, not log

Uppercase letters:

upper, followed by name of letter

Uppercase word:

upword, followed by sequence of letters in word, pronounced one letter at a time

Greek letters:

Greek, followed by English name of letter; example, Greek e
Greek name; example, epsilon
provide readers with reference card containing upper and lowercase Greek symbols and names

Uppercase Greek letters:

Greek upper e
upper epsilon

Italic/bold/script/sanserif/underlined:

never developed protocol; has not been necessary

Digits:

always pronounce numbers individually, never as words
examples: 1 5, not fifteen; 1 0 0, not one hundred

Punctuation:

comma, any position: comma
decimal point, any position: point
period: period
colon: colon

Longer punctuation: abbreviated forms are used for the following:

semicolon: semi
exclamation point: shriek
grouping symbols:
( ) right/left parenthesis: R-pare, L-pare
{ } right/left bracket: R-brack, L-brack
[ ] right/left brace: R-brace, L-brace
< > right/left angle bracket: R-angle, L-angle

Operators and other mathematics symbols:

+ plus
- minus
• multiplication dot: dot
x multiplication cross: cross

* asterisk: star
/ slash: slash
⊃ left-opening horseshoe, set-theoretic context: upperset
⊃ left opening horseshoe, logical context: implies
⇒ right pointing arrow with double horizontal line: implies
⊂ right-opening horseshoe: subset
∪ up-opening horseshoe: cup (meaning union)
∩ down-opening horseshoe: cap (meaning intersection)
< right-opening wedge: less
> left-opening wedge: greater
∨ up-opening wedge: join
∧ down-opening wedge: meet
≤ modified right-opening wedge: less-equal, or not less
≥ modified left-opening wedge: greater-equal, or not greater
= equals
≠ canceled-out equals sign: not equal
∈ set notation graphic: element
∋ reverse of set notation graphic: contains
∂ round d: partial
∇ inverted uppercase delta: del
$ slashed s: dollar
¢ slashed c: cent
! integral sign: integral
∞ infinity sign: infinity
∅ slashed O: empty set
° small elevated circle: degree
% percent sign: percent
& ampersand sign: ampersand
_ underbar sign: underbar
number sign or pound sign: crosshatch
clear space in print: space

Fractions:

begin fraction: B-frac
end fraction: E-frac
fraction line: over
examples: one-half: B-frac 1 over 2 E-frac
B-frac a plus b over c plus d E-frac

simple fraction (no subsidiary fractions): of order 0

complex fraction (at least one subsidiary fraction): of order 1

example: B-B-frac, O-over, E-E-frac

hypercomplex fraction: of order 2

example: B-B-B-frac, O-O-over, E-E-E-frac

Radicals:

treated much like fractions

beginning of radical: B-rad

end of radical: E-rad

example: square root of 2 is B-rad 2 E-rad

nested radicals: treated like nested fractions, except no corresponding component for over

example: B-B-rad a plus B-rad a plus b E-rad plus b E-E-rad

Subscripts and superscripts:

subscript: sub

superscript: sup (pronounced like soup)

example: x sup 2, not x square

return to base level: base

example: Pythagorean Theorem is spoken as z sup 2 base equals x sup 2 base plus y sup 2 base period

change in level: path is spoken, beginning at base level and ending at new level

example: if e has a superscript of x, x has a subscript of I + j, we would say e sup x sup-sub I plus j

example: if e has a superscript of x, x has a superscript of 2, we would say e sup x sup-sup 2

element carrying both subscript and superscript: speak all of the subscript first, then all of the superscript

example: if e has a superscript of x, and x has a subscript of I + j and a superscript of p sub k, we would say e sup x sup-sub I plus j sup-sup p sup-sup-sup k

radical other than the square root:

speak the radical index as a superscript to the radical

example: the cube root of x + y is B-rad sup 3 base x lqs y E-rad

first level underscript: underscript

first level overscript: overscript

termination of underscript or overscript: endscript

example: upper sigma underscript I equals 1 overscript n endscript a sub I.

second level underscript: un-underscript

second level overscript: o-overscript

order of underscripts and overscripts: speak all underscripts in the order of descending level before speaking any overscripts; precede each level by speaking underscript with the proper number of un prefixes attached; speak all overscripts in order of ascending level; precede each level with overscript, with the proper number of prefixes attached

APPENDIX A

MATHEMATICS AND THE BLIND STUDENT

(taken from: *NEW BEACON*, Vol. XVIII, No. 210,
June 15, 1934, pp. 146–148)

MATHEMATICS AND THE BLIND STUDENT

There are two essentials for the blind student of mathematics. The first is a comprehensive system of notation, capable of expressing all mathematical relationships neatly and concisely, for until a system is devised the student is obliged to improvise his own method, and such improvisation is often clumsy and apt to prove incapable of expressing all the niceties required of it. The second is apparatus, primarily to take the place of the pencil and paper which enables the seeing student of such a subject as geometry to draw the picture of the problem that he seeks to solve, and so to have something concrete before him.

Although it is true that the higher mathematician more and more dispenses with the concrete as he comes to move in those realms that are not far removed from philosophy, there is a long and arduous journey to be travelled before such heights are reached, and on that journey the blind student needs apparatus as truly as the seeing. If the blind lover of mathematics persists, it is possible that in time he may be more at home in these higher reaches of mathematics than his seeing rivals, and may dispense even more readily with external aids ("Geometry is the proper science for the blind because no help is needed to carry it to perfection," said an eighteenth century blind mathematician), but such heights are attainable only to a chosen few.

There have been great blind mathematicians in the past, long before any recognised system of notation for the blind student existed, notably of course Nicholas Saunderson, Lucasian Professor of Mathematics at Cambridge in the early eighteenth century. He worked out his arithmetical, algebraical, and geometrical problems on a square board, itself divided into smaller squares, with a hole in each square in which Saunderson placed pegs. Less outstanding than Saunderson, but worthy of mention, was Herr Weissenberg (born 1756), who was fortunate in having a very enterprising tutor, who taught him algebra, trigonometry, and geometry, and who modified Saunderson's board for his young pupil, introducing certain improvements. Later on, the board was still further modified by Valentine Hauy, of the Institution des Jeunes Aveugles.

And lest we should be tempted to think of skill in mathematics as a purely masculine achievement, we read of Mademoiselle de Salignac (born 1741), whose conversation with Diderot is thus described by Diderot himself: "I said one day to her: 'Mademoiselle, figure to yourself a cube,' 'I see it,' she said. 'Imagine a point in the centre of the cube.' 'It is done.' 'From this point draw lines directly to the angles: you will then have divided the cube—' 'Into six equal pyramids,' she answered, 'having every one the same faces: the base of the cube and the half its height.'"

In face of this erudition, it is comforting to read later that Mademoiselle de Salignac was not lacking in more feminine graces, for it is stated that she made

"garters, bracelets, and collars for the neck, with very small glass beads sewed upon them in alphabetical patterns."

One of the outstanding figures in the history of the early education of the blind in this country was the Rev. William Taylor, first Superintendent, nearly a hundred years ago, of the Wilberforce Memorial School, York, and later one of the founders of the College and the Blind Sons of Gentlemen at Worcester. He is remembered in schools for the blind today as the inventor of the Taylor Arithmetic frame, with its star-shaped eight-angled holes, and metal type. For many years the Taylor frame was the only piece of apparatus used for the teaching of mathematics, but because, in spite of its undoubted ingenuity, it is rather a cumbrous appliance, comparing very unfavourably with the pencil and paper of the seeing mathematician, it was only rarely that the blind boy or girl progressed further than a working knowledge of elementary arithmetic. Even today, most blind people of average education will admit that when they leave school days behind them they also discard the Taylor frame, though it is hoped that the recently devised cover for the frame, which enables the board to be carried about without disarranging the type, may make it of more practical service.

As we have said above, various systems of notation have been devised from time to time by the blind student of mathematics, and by the teachers in various schools, but for many years there was no uniformity in this respect, so that the sign which for one student stood for *plus* might conceivably stand for *minus* somewhere else. One Braille notation was devised by the eminent Cambridge mathematician, Henry Martyn Taylor, who was overtaken by blindness in 1894, when engaged in the preparation of an edition of Euclid for the Cambridge University Press. By means of his ingenious and well thought out Braille notation he was enabled to transcribe many advanced scientific and mathematical works, and in 1917, with the assistance of Mr. Emblen, a blind member of the staff of the National Institute for the Blind, he perfected it. It was recognised as so comprehensive that it was soon adopted as the standard mathematical and chemical notation, and is universally used by English-speaking people.

In 1914, Mr. G. B. Brown, himself a mathematician, was appointed Principle of Worcester College for the Blind, and his enthusiasm infected his pupils, so that under his direction they improved the school apparatus, and among other things devised a graph board, enabling them to do algebraical and trigonometrical graphs.

A few years later, when Mr. Emblen was engaged in coaching Miss Sadie Isaacs (a brilliant blind girl who in 1924 took her London degree with honours, and was awarded a scholarship as the student who gained first place in the University), he was brought forcibly up against the lack of apparatus for the blind student of mathematics. The toothed wheel pencil and compasses, enabling pupils to make their own geometrical figures, had been invented many years before by Mr. Guy Campbell, but otherwise there was little available. As a result, Mr. Emblen invented a mathematical demonstra-

tion board, which is now very generally used for the study of geometry and the plotting of graphs. It consists of a baize-covered board, marked in half-inch and centimetre squares, and into it pins are inserted. Geometrical figures, such as the triangle or parallelogram, are made by slipping rubber bands over these pins, while circles are made by means of flexible steel bands, slotted at one end to allow the insertion of the other; quite elaborate figures, such as the nine-point circle, can be rapidly and easily made by means of this board.

About 1918, Mr. Taylor introduced algebra type for use with the Taylor Arithmetic frame (the invention of his namesake many years before), and together he and Mr. Emblen compiled a pamphlet "How to write Arithmetic and Algebra by means of the Joint Type Method." This is a companion volume to Mr. Emblen's "Guide to the writing of Arithmetic and Algebra, with Mathematical and Chemical Formulae," a study of which will enable the Braillist, whether he is a mathematician or not, to transcribe into Braille any scientific or mathematical book. The fact that books as widely varying as Godfrey and Bell's "Winchester Arithmetic," Godfrey and Siddons' "Elementary Algebra," Darwin's "Tides and Other Phenomena of the Solar System," Marr's "Introduction to Geology," Ashford's "Electricity and Magnetism," Fletcher's "Elements of Plane Trigonometry," Smith's "Conic Sections," Eggar's "Mechanics," Jeans' "Universe Around Us" (all illustrated with diagrams, where these are to be found in the ink-print versions) have been published in Braille, is an indication that the system of mathematical and chemical notation devised by Mr. Taylor and Mr. Emblen is able to meet the very heavy strain laid upon it, and can justly claim to be comprehensive.

More recently, Mr. Emblen has been responsible for a List of Tables of Weights and Measures, with the metrical equivalent in every case given on the same line of Braille, and for "A Text Book of Mathematical Tables," including 4 figure logarithms, trigonometrical ratios, and various formulae.

An International Committee was appointed at the Vienna Conference in 1929 whose aim it is to secure uniformity of mathematical and scientific notation. The English representative on this Committee is Colonel Stafford, who, while he is keenly alive to English interests, is even more keenly alive to the importance of securing a measure of uniformity, if it can be done for the mutual good of all. The task of the Committee is not an easy one, as no country every lightly discards the system it has adopted as its own; but just as uniformity in Braille music notation has broken down frontiers, and brought the music of many nations within the reach of all those who have adopted the code, so it is hoped that a similar measure of uniformity may be achieved in the realm of mathematics and science.

APPENDIX B

HANDBOOK FOR SPOKEN MATHEMATICS

(Larry's Speakeasy)

HANDBOOK FOR

SPOKEN MATHEMATICS

(Larry's Speakeasy)

Lawrence A. Chang, Ph.D.

With assistance from
Carol M. White
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CONTENTS

HANDBOOK FOR SPOKEN MATHEMATICS

Section 1—Introduction

This handbook answers some of the needs of the many people who have to deal with spoken mathematics, yet have insufficient background to know the correct verbal expression for the written symbolic one. Mathematical material is primarily presented visually, and when this material is presented orally, it can be ambiguous. While the parsing of a written expression is clear and well-defined, when it is spoken this clarity may disappear. For example, "One plus two over three plus four" can represent the following four numbers, depending on the parsing of the expression: 3/7, 12/7, 5, 5 2/3. However, when the corresponding written expression is seen, there is little doubt which of the four numbers it represents. When reading mathematics orally, such problems are frequently encountered. Of course, the written expression may always be read symbol by symbol, but if the expression is long or there are a cluster of expressions, it can be very tedious and hard to understand. Thus, whenever possible, one wishes to have the written expression spoken in a way that is interest retaining and easy to understand.

In an attempt to alleviate problems such as these, this handbook has been compiled to establish some consistent and well-defined ways of uttering mathematical expressions so that listeners will receive clear, unambiguous, and well-pronounced representations of the subject.

Some of the people who will benefit from this handbook are: 1) those who read mathematics orally and have insufficient background in the subject, and their listeners; 2) those interested in voice synthesis for the computer, particularly those who deal with spoken symbolic expressions; and 3) those technical writers and transcribers who may need to verbalize mathematics.

This edition of the handbook is a working one, and it is hoped that the people who use it will add to and refine it. The choice of material and its ordering are my own preferences, and, as such, they reflect my biases. A goal of the handbook is to establish a standard where no standard has existed, so far as I know. However, this standard represents only one of many possibilities. As a blind person, I have learned mathematics by means of others reading the material to me; so my preferences are a result of direct experience.

This handbook is organized as follows: In Section II the various types of alphabets used in mathematics are listed. Section III lists the basic symbols used in mathematics, along with their verbalizations. Sections IV-XI list the expressions used in some of the more common branches of mathematics, along with their verbalizations. Section XII contains some suggestions on how to and how not to describe diagrams.

To use this handbook efficiently, it is suggested that you look over Sections II and III or alphabets and basic symbols. Next, establish which section most closely relates to the subject matter at hand. There may also be material in other sections that you can use if you cannot find what you need in the related section. In many sections, more than one choice for a given expression is offered to the user. Once the choice has been made, the reader should use it consistently throughout the text. If you encounter an expression that is not included in the guide, read the expression literally, that is, read it from left to right, symbol by symbol.

For those who are interested in speech synthesis and speech recognition for the computer, this handbook may provide some basic ideas and suggestions regarding the formulation of spoken mathematics. With speech synthesis, when the computer reads a file containing many mathematical expressions, the speech synthesizer will speak the expressions symbol by symbol. As we have pointed out before, this process can be tedious and hard to understand. A program that could translate the mathematical expressions from the symbol to symbol form into a spoken form that is more intelligible can ease the task for those who use synthetic speech. On the other hand, if one wishes to communicate mathematical expressions to the computer by voice, a program that will translate spoken expressions of mathematics into written expressions with the correct parsing is essential. This handbook provides a basis for writing these programs, both for speech synthesis and voice recognition, by giving examples of written mathematical expressions followed by the word for word spoken form of the same expressions. An example where these ideas are of particular relevance is the voice input and output of computer programs that manipulate symbolic expressions, because both the input and output of the program are mathematical expressions.

I would like to thank the Office of Equal Opportunity of the Lawrence Livermore National Laboratory for their support in bringing this handbook into fruition. Thanks also go to my wife for her untiring help and to my friends and colleagues at the Lab for their assistance.

October, 1983 Lawrence Chang

Section II—Alphabets

Roman Alphabet

Read capital or upper-case letters as capital *lettername* or cap *lettername*. Read small or lower-case letters as small *lettername*.

Capital or upper-case:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Small or lower-case:
a b c d e f g h i j k l m n o p q r s t u v w x y z

Types of Roman Alphabets

Italic: Read capital or upper-case letters as italic capital *lettername*. Read small or lower-case letters as italic *lettername*.

Capital or upper-case:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Small or lower-case:
a b c d e f g h i j k l m n o p q r s t u v w x y z

Boldface: Read capital or upper-case letters as boldface capital *lettername*. Read small or lower-case letters as boldface *lettername*.

Capital or upper-case:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Small or lower-case:
a b c d e f g h i j k l m n o p q r s t u v w x y z

Gothic or Old English

Read capital or upper-case letters as Gothic capital *lettername*. Read small or lower-case letters as Gothic *lettername*.

Capital or upper-case:
𝔄 𝔅 ℭ 𝔇 𝔈 𝔉 𝔊 ℌ ℑ 𝔍 𝔎 𝔏 𝔐 𝔑 𝔒 𝔓 𝔔 ℜ 𝔖 𝔗 𝔘 𝔙 𝔚 𝔛 𝔜 ℨ

Corresponding Roman-letter:
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Small or lower-case:
𝔞 𝔟 𝔠 𝔡 𝔢 𝔣 𝔤 𝔥 𝔦 𝔧 𝔨 𝔩 𝔪 𝔫 𝔬 𝔭 𝔮 𝔯 𝔰 𝔱 𝔲 𝔳 𝔴 𝔵 𝔶 𝔷

Script

Read capital or upper-case letters as script capital *lettername*. Read small or lower-case letters as script *lettername*.

Capital or upper-case:
$\mathscr{A}\,\mathscr{B}\,\mathscr{C}\,\mathscr{D}\,\mathscr{E}\,\mathscr{F}\,\mathscr{G}\,\mathscr{H}\,\mathscr{I}\,\mathscr{J}\,\mathscr{K}\,\mathscr{L}\,\mathscr{M}\,\mathscr{N}\,\mathscr{O}\,\mathscr{P}\,\mathscr{Q}\,\mathscr{R}\,\mathscr{S}\,\mathscr{T}\,\mathscr{U}\,\mathscr{V}\,\mathscr{W}\,\mathscr{X}\,\mathscr{Y}\,\mathscr{Z}$

Small or lower-case:
a b c d e f g h i j k l m n o p q r s t u v w x y z

Greek Alphabet

Read capital or upper-case letters as capital *lettername* or cap *lettername*. Read small or lower-case letters as small *lettername*.

Capital	Small	Name	Pronunciation	Corresponding Roman letter
Α	α	alpha	al′ fuh	a
Β	β	beta	bay′ tuh	b
Γ	γ	gamma	gam′ muh	g
Δ	δ	delta	del′ tuh	d
Ε	ε	epsilon	ep′ suh lon	e
Ζ	ζ	zeta	zay′ tuh	z
Η	η	eta	ay′ tuh	ē
Θ	θ	theta	thay tuh	th
Ι	ι	iota	i oh tuh	i
Κ	κ	kappa	kap′ puh	k
Λ	λ	lambda	lam′ duh	l
Μ	μ	mu	mew	m
Ν	ν	nu	new	n
Ξ	ξ	xi	zigh or ksigh	x
Ο	ο	omicron	om′ uh cron	o
Π	π	pi	pie	p
Ρ	ρ	rho	row (as in rowboat)	r, rh
Σ	σ, s	sigma	sig′ muh	s
Τ	τ	tau	tow (rhymes with cow)	t
Υ	υ	upsilon	up′ suh lon	y, u
Φ	φ	phi	fi (rhymes with hi)	ph
Χ	χ	chi	ki (rhymes with hi)	ch
Ψ	ψ	psi	sigh or psigh	ps
Ω	ω	omega	oh meg′ uh	ō

SECTION III – BASIC SYMBOLS

Symbol		Speak	Notes
$+$	or	plus positive	
$-$	or	minus negative	
x $\cdot$	or	multiples times	
$\div$ $/$		divided by	
$\lvert\ \rvert$		absolute value	
$\mid$		divides	
$\pm$		plus or minus	
$\mp$		minus or plus	
$\oplus$		circle plus	
$\otimes$		circle cross	
$=$	or	equals equal to	
$\neq$	or	does not equal not equal to	
$\equiv$		identical to	
$\not\equiv$		not identical to	
$\approx$ $\simeq$		approximately equal to	
$\sim$		equivalent to	
$\lesssim$		approximately equal but less than	
$\leq$		less than or equal to	

Symbol		Speak	Notes
$<$		less than	
$\ll$		much less than	
$\nless$		not less than	
$\gtrsim$		approximately equal but greater than	
$\geq$		greater than or equal to	
$>$		greater than	
$\gg$		much greater than	
$\ngtr$		not greater than	
(	or	open parenthesis left parenthesis	
)	or	closed parenthesis right parenthesis	
[	or	open bracket left bracket	
]	or	closed bracket right bracket	
{	or	open brace left brace	
}	or	closed brace right brace	
—		vinculum	Example: $a-\overline{b-c}$ is read as a minus vinculum b minus c.

In the next examples, the letter a is used with the symbol for clarity – the letter a is a dummy variable.

Symbol		Speak	Notes
$\lvert a \rvert$		absolute value of a	In this case a is any real number.
a'		a prime	If a is an angle, a′ is read as a minutes.
a''		a double prime	If a is an angle a″ is read as a seconds.
$a^{[n]}$		a with n primes	
a^n	or	a superscript n a to the n	
$\bar{a}$		a bar	
a^*	or	a star a super asterisk	
a_n	or	a subscript n a sub n	When n = 0, a_n may be read as a naught.
$\sqrt{}$		radical sign	
$\sqrt{a}$		square root of a	
$\sqrt[3]{a}$		cube root of a	
$\sqrt[n]{a}$		nth root of a	
$\varnothing$	or	zero null set	to distinguish from the letter o

Symbol		Speak	Notes
ƶ		the letter z	to distinguish from 2
א		ah′ lef	aleph, the first letter of the Hebrew alphabet
$\prod$		product	Example: $\prod_{i=1}^{n}$ is read product from i=1 to n.
$\sum$		summation	Example: $\sum_{i=1}^{n}$ is read summation from i=1 to n
$\int$		integral	Example: $\int_a^b$ is read integral from a to b
d/dx	or or	d over d x d by d x the derivative with respect to x	
∂/∂x	or	the partial derivative with respect to x partial over partial x	
∇		del	
!		factorial	Example: n! is read n factorial
*	or	star asterisk	
&	or	ampersand and	
†		dagger	

Symbol		Speak	Notes
††		double dagger	
a^0		a degrees	
$a^{(r)}$		a radians	
§		section	
$\parallel$		parallel	
$\perp$		perpendicular	
$<$, $\measuredangle$, $\angle$		angle	
$\llcorner$		right angle	
Δ		triangle	However, Δx is read delta x or increment x.
▱		parallelogram	
□		square	
○		circle	
⬭	or	ellipse oval	
⌒		arc	Example: $\overset{\frown}{AB}$ is read arc ab.
$\therefore$		therefore	
$\because$	or	since because	
$\cdots$	or or	dot, dot, dot ellipsis etc.	
:	or	is to ratio	

Symbol		Speak	Notes
$::$	or	as proportion	
$\wedge$	or	hat circumflex	Example: $\hat{a}$ is read a hat or a circumflex.
¨		ooḿ laut	Example: ä is read a ooḿ laut
`		accent grave	Example: à is read a accent grave.
´		accent acute	Example: á is read a accent acute.
~		til duh	Example: ñ is read n til duh.
$\wedge$		caret	
$\rightarrow$	or	arrow to the right approaches	
$\leftarrow$	or	arrow to the left withdraws	
$\uparrow$	or	arrow pointing up upward arrow	
$\downarrow$	or	arrow pointing down downward arrow	
$\vec{a}$		vector a	
$\cup$, $\vee$		union	

Symbol		Speak	Notes
$\cap$		intersection	
$\subset$	or	contained in subset of	
$\supset$		contains	
$\Rightarrow$		implies	
$\Leftrightarrow$		equivalent to	
iff		if and only if	
$\exists$	or	there exists there is	
$\forall$		for every	
$\ni$		such that	
%		percent	
$		dollars	
¢		cents	
@		at	
#	or or	sharp pound sign number sign	
♭		flat	
$\propto$		proportional to	
∞		infinity	

SECTION IV — ALGEBRA

The small letters of the alphabet, a, b, c, d, ..., may be any numbers.

Expression		Speak	Notes
a + b		a plus b	
a + b + c		a plus b plus c	
a - b		a minus b	
– a – b		minus a minus b	
a + b – c		a plus b minus c	
a – b – c		a minus b minus c	
a – (b + c)	 or or	a minus the sum b plus c a minus the quantity b plus c a minus open parenthesis b plus c close parenthesis	
a – (b – c)	 or or	a minus the difference b minus c a minus the quantity b minus c a minus open parenthesis b minus c close parenthesis	
a–(–b–c)	 or	a minus the quantity minus b minus c a minus open parenthesis minus b minus c close parenthesis	
a – (b + c) – d	 or	a minus the quantity b plus c end of quantity minus d a minus open parenthesis b plus c close parenthesis minus d	
a – b – (c – d)	 or or	a minus b minus the difference c minus d a minus b minus the quantity c minus d a minus b minus open parenthesis c minus d close parenthesis	

Expression		Speak	Notes
a X b		a times b	
	or	a cross b	
	or	the product of a and b	
	or	a multiplied by b	
$a \cdot b$		a times b	
	or	a dot b	
	or	the product of a and b	
	or	a multiplied by b	
ab		a b	
	or	a times b	
	or	the product of a and b	
	or	a multiplied by b	
$a \cdot -b$		a times minus b	
ab + c		a b plus c	
a(b + c)		a times the sum b plus c	
	or	a times the quantity b plus c	
	or	a times open parenthesis b plus c close parenthesis	
a(b + c) + d		a times the quantity b plus c end of quantity plus d	
	or	a open parenthesis b plus c close parenthesis plus d	
ab – c		a b minus c	
a (b – c)		a times the difference b minus c	
	or	a times the quantity b minus c	
	or	a open parenthesis b minus c close parenthesis	
a(–b – c)		a times the quantity minus b minus c	
	or	a open parenthesis minus b minus c close parenthesis	

Expression		Speak	Notes
$a(b - c + d)$		a times the quantity b minus c plus d	
	or	a open parenthesis b minus c plus d close parenthesis	
$ab + cd$		a b plus c d	
$ad - bc$		a d minus b c	
$a(b + c) - e\,(f - g)$		a times the quantity b plus c end of quantity minus e times the quantity f minus g	
	or	a open parenthesis b plus c close parenthesis minus e open parenthesis f minus g close parenthesis	
$a[b + c - e(f - g)]$		a times the quantity b plus c minus the product e times the difference f minus g end of quantity	
	or	a open bracket b plus c minus e open parenthesis f minus g close parenthesis, close bracket	
$(a + b)(c + d)$		the sum a plus b times the sum c plus d	
	or	the product of the sum a plus b and the sum c plus d	
	or	open parenthesis a plus b close parenthesis open parenthesis c plus d close parenthesis	
$\frac{1}{2}$		one half	
	or	one over two	
$\frac{1}{3}$		one third	
	or	one over three	
$\frac{1}{n}$		one over n	

Expression		Speak	Notes
$\frac{a}{d}$, a/d, $a \div d$		a over d	
	or	a divided by d	
	or	the ratio of a to d	
$\frac{a+b}{d}$		the fraction, the numerator is a plus b the denominator is d	
	or	the quantity a plus b divided by d	
$a + \frac{b}{d}$		a plus the fraction b over d	
$a + \frac{b}{c+d}$		a plus the fraction, the numerator is b and the denominator is c plus d	
	or	a plus the fraction b divided by the quantity c plus d	
$\frac{a+b}{c} + d$		the quantity a plus b over c, that fraction plus d	
$a + \frac{b}{c} + d$		a plus the fraction b over c, that fraction plus d	
$\frac{a}{b} + \frac{c}{d}$		the fraction a over b plus the fraction c over d	
$\frac{a}{b + \frac{c}{d}}$		the fraction, the numerator is a, the denominator is the sum b plus the fraction c over d	
$\frac{\frac{a}{b}}{d}$		the fraction, the numerator is the fraction a over b, the denominator is d	
	or	a over b, that fraction divided by d	

Expression		Speak	Notes
$\dfrac{a}{\frac{c}{d}}$		a divided by the fraction c over d	
$\dfrac{\frac{a}{b}}{\frac{c}{d}}$	or	the fraction, the numerator is the fraction a over b, the denominator is the fraction c over d the fraction a over b divided by the fraction c over d	
$\dfrac{\frac{a+b}{c}}{d}$	or	the fraction, the numerator is the quantity a plus b over c, the denominator is d the quantity a plus b over c, that fraction divided by d	
$\frac{c}{d}(a+b)$		the fraction c over d times the sum a plus b	
$\dfrac{a}{\frac{b}{c+d}}$		a divided by the fraction b over the quantity c plus d	
$a(b+\frac{c}{d})$		a times the sum b plus the fraction c over d	
$a+\cfrac{b}{a+\cfrac{b}{a+\cfrac{b}{a+\cfrac{b}{\vdots}}}}$		the continued fraction: a plus the fraction b divided by the sum a plus the fraction b divided by the sum a plus the fraction b divided by the sum a plus the fraction b divided by dot dot dot	

Expression		Speak	Notes
$ay + bx + c = 0$		a y plus b x plus c equals zero	linear equation
$y = mx + b$		y equals m x plus b	
$y = ax^2 + bx + c$		y equals a x squared plus b x plus c	quadratic equation
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	or	x equals minus b plus or minus the square root of the difference b squared minus 4 a c, that whole quantity divided by 2 a x equals the fraction, the numerator is minus b plus or minus square root of the difference b squared minus 4 a c, the denominator is 2 a	
$x^2 + y^2 = r^2$		x squared plus y squared equals r squared	circle with radius r, center at origin
$y = \pm\sqrt{r^2 - x^2}$		y equals plus or minus square root of the difference r squared minus x squared	respectively, upper or lower semicircle with radius r, center at origin
$(x-h)^2 + (y-k)^2 = r^2$	or	the difference x minus h squared plus the difference y minus k squared equals r squared the quantity x minus h squared plus the quantity y minus k squared equals r squared	
$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$		the fraction x squared over a squared plus the fraction y squared over b squared equals 1	ellipse
$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$		the fraction x squared over a squared minus the fraction y squared over b squared equals 1	hyperbola

Symbol		Speak	Notes
$ax^2 + bxy + cy^2 + dx + ey + f = 0$		a x squared plus b x y plus c y squared plus d x plus e y plus f equals zero	the conics
a^x	or	a to the x a raised to the x power	
e^{x+y}	or	e to the quantity x plus y power e raised to the x plus y power	
$e^x + y$	or	the sum of e to the x and y e to the x power plus y	
$e^x e^y$		the product of e to the x power and e to the y power	
e^{xa^y}	or	e raised to the x times a to the y power e raised to the product of x and a to the y	
$e^x y$	or	the product of e to the x power and y e raised to the x power times y	
$e^{i2\pi z}$	or	e to the quantity i 2 pi z power e raised to the i 2 pi z power	
$\log_b a$		log to the base b of a	
$\log_{10} 3 \cdot 4$		log to the base 10 of the product 3 times 4	
$\log_e \frac{2}{5}$	or	log to the base e of the fraction 2 over 5 log to the base e of the ratio 2 to 5	
$\ln x$	or	the natural log of x l n of x	

Expression		Speak	Notes
$a_1 + a_2 + \ldots + a_n$		a sub 1 plus a sub 2 plus dot dot dot plus a sub n	
	or	a sub 1 plus a sub 2 plus ellipsis plus a sub n	
$a_1 \cdot a_2 \cdot \ldots \cdot a_n$		a sub 1 times a sub 2 times dot dot dot times a sub n	
	or	a sub 1 times a sub 2 times ellpisis times a sub n	
$a_1 \cdot b_1 + a_2 \cdot b_2 + \ldots + a_n \cdot b_n$		a sub 1 times b sub 1 plus a sub 2 times b sub 2 plus dot dot dot plus a sub n times b sub n	
	or	a sub 1 times b sub 1 plus a sub 2 times b sub 2 plus ellipsis plus a sub n times b sub n	
$p(x)$		p of x	In algebra
$p(x) = 3x^2 + 2x - 4$		p of x equals 3 x squared plus 2 x minus 4	
$q(x) = x^3 - 8$		q of x equals x cubed minus 8	
$p(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n$		p of x equals a sub zero x to the n plus a sub 1 x to the n minus 1 plus dot dot dot plus a sub n minus 1 x plus a sub n	polynomial of n degree
	or	p of x equals a sub zero times x raised to the n power plus a sub 1 times x to the n minus 1 power plus dot dot dot plus a sub the quantity n minus 1 times x plus a sub n	for extra clarity, when in doubt

SECTION V – TRIGONOMETRIC AND HYPERBOLIC EXPRESSIONS

The Greek letter θ (theta) will be used in this section to denote an angle in degrees or radians.

Expressions	Speak	Notes
θ	theta degrees	
θ'	theta minutes	
θ''	theta seconds	
s.a.s.	side angle side	
s.s.s.	side side side	

The six basic trigonometric functions are:

Function		Speak	Notes
$\sin\theta$	or	sine of theta sine theta	
$\cos\theta$	or	co sine of theta co sine theta	
$\tan\theta$	or	tangent of theta tangent theta	
$\cot\theta$	or	co tangent of theta co tangent theta	
$\sec\theta$	or	see cant of theta see cant theta	
$\csc\theta$	or	co see cant of theta co see cant theta	

Other functions are:

Function		Speak	Notes
$\sin^2 x$		sine squared x	
$\cos^2 x$		co sine squared x	
$\tan^2 x$		tangent squared x	
$\cot^2 x$		co tangent squared x	
$\sec^2 x$		see cant squared x	
$\csc^2 x$		co see cant squared x	
$\sinh\ \theta$	or	hyperbolic sine theta sinch theta	
$\cosh\ \theta$	or	hyperbolic co sine theta cosh theta	
$\tanh\ \theta$	or	hyperbolic tangent theta tange theta	
$\coth\ \theta$		hyperbolic co tangent theta	
$\operatorname{sech}\ \theta$		hyperbolic see cant theta	
$\operatorname{csch}\ \theta$		hyperbolic co see cant theta	
$\left.\begin{array}{r}\text{arc sin } x\\ \sin^{-1} x\end{array}\right\}$	or or or	arc sine x inverse sine x anti sine x sine to the minus 1 of x	The negative exponent does not mean the reciprocal of the function n nor $\frac{1}{\text{the function}}$
$\left.\begin{array}{r}\text{arc cos } x\\ \cos^{-1} x\end{array}\right\}$	or or or	arc co sine x inverse co sine x anti co sine x co sine to the minus 1 of x	

Function		Speak	Notes
arc tan x $\tan^{-1} x$	 or or or	arc tangent x inverse tangent x anti tangent x tangent to the minus 1 of x	
arc cot x $\cot^{-1} x$	 or or or	arc co tangent x inverse co tangent x anti co tangent x co tangent to the minus 1 of x	
arc sec x $\sec^{-1} x$	 or or or	arc see cant x inverse see cant x anti see cant x see cant to the minus 1 of x	
arc csc x $\csc^{-1} x$	 or or or	arc co see cant x inverse co see cant x anti co see cant x co see cant to the minus 1 of x	
arc sinh x $\sinh^{-1} x$	 or or or	arc hyperbolic sine of x arc sinch x inverse hyperbolic sine of x anti hyperbolic sine of x	
arc cosh x $\cosh^{-1} x$	 or or or	arc hyperbolic co sine of x arc cosh x inverse hyperbolic co sine of x anti hyperbolic co sine of x	

Function		Speak	Notes
arc tanh x		arc hyperbolic tangent of x	
$\tanh^{-1} x$	or	arc tange x	
	or	inverse hyperbolic tangent of x	
	or	anti hyperbolic tangent of x	
arc coth x		arc hyperbolic co tangent of x	
$\coth^{-1} x$		inverse hyperbolic co tangent of x	
	or	anti hyperbolic co tangent of x	
arc sech x		arc hyperbolic see cant of x	
$\mathrm{sech}^{-1} x$	or	inverse hyperbolic see cant of x	
	or	anti hyperbolic see cant of x	
arc csch x		arc hyperbolic co see cant of x	
$\mathrm{csch}^{-1} x$	or	inverse hyperbolic co see cant of x	
	or	anti hyperbolic co see cant of x	

The following expressions can be used for any of the six trigonometric functions: sine cosine, tangent, cotangent, secant, cosecant. In the examples that follow, sine will be used.

Function		Speak	Notes
$\sin \theta + x$		sine of theta, that quantity plus x	
$\sin (\theta + \omega)$		sine of the sum theta plus omega	
	or	sine of the quantity theta plus omega	
$(\sin \theta)\, x$		sine theta times x	
$\sin (\theta\omega)$		sine of the product theta omega	
$(\sin \theta^2)\, x$		sine of theta squared, that quantity times x	
$\sin^2 \theta \cos \theta$		sine squared theta times co sine theta	
$\sin\theta \cos\theta$		sine of theta times co sine of theta	
$\sin (\theta \cos\theta)$		sine of the product theta times co sine theta	

SECTION VI — LOGIC AND SET THEORY

Expression		Speak	Notes
$\therefore$		therefore	
$\ni$		such that	
$\sim p$ $-p$ $\bar{p}$ p'		not p	The reader must be careful to differentiate between $\tilde{p}$ (p tilde) and $\bar{\ }p$ (not p).
$p \wedge q$ $p \cdot q$ $p \& q$	or	both p and q p and q	
pVq $p \vee q$	or	at least one of p and q p or q	
$p\vert q$ p/q	or	not both p and q not p or not q	
$p \downarrow q$ $p \Delta q$		neither p nor q	
$p \therefore q$ $p < q$	or	if p, then q p only if q	
$\leftrightarrow$ $.:\ :.$ $\equiv$ $\sim$ iff		if and only if	
V **V** **I** I		universal class	
ϕ Λ Λ 0		null class	

Expression		Speak	Notes
$\left.\begin{array}{l}(x)\\ A_x\\ \forall_x\end{array}\right\}$		for all x	
$\left.\begin{array}{l}A_{x,y,}\cdots\\ \forall_{x,y,}\cdots\end{array}\right\}$	or	for all x, y, ellipsis for all x, y, dot dot dot	
$\exists$		there exists	
$\left.\begin{array}{l}(\exists x)\\ Ex\\ \Sigma_x\end{array}\right\}$		there is an x such that	
$E_{x,y,\cdots}$		there exists x, y, dot dot dot such that	
$\left.\begin{array}{l}E_x\\ C_x\end{array}\right\}$		the class of all objects x that satisfy the condition	Example: $E_x(x-a)<0$ is read the class of all objects x that satisfy the condition the quantity x minus a is less than zero.
$\left.\begin{array}{l}[x\|s(x)]\\ [x:s(x)]\end{array}\right\}$		the class of all objects x which satisfy s of x	

Note: In the following expressions the capital letters M, N, and P denote sets.

$\left.\begin{array}{l}x\in M\\ x\in M\end{array}\right\}$	or	x is an element of the set capital m the point x belongs to the set capital m	
$M\subset N$	or	capital m is a subset of capital n capital m is contained in capital n	
$M\subseteq N$		capital m is a subset of or equal to capital n	
$M\supset N$		capital m contains capital n	
$M\supseteq N$		capital m contains or is equal to capital n	
$\left.\begin{array}{l}M\cap N\\ M\cdot N\end{array}\right\}$		intersection of capital m and capital n	

Expression		Speak	Notes
$\left.\begin{array}{l} M \cup N \\ M + N \end{array}\right\}$	or or	union of capital m and capital n join of capital m and capital n sum of capital m and capital n	
$\left.\begin{array}{l} \cap_{\alpha \varepsilon A} M_\alpha \\ \Pi_{\alpha \varepsilon A} M_\alpha \end{array}\right\}$	or	intersection of all the sets capital m sub alpha with alpha an element of capital a product of all the sets capital m sub alpha with alpha an element of capital a	
$\left.\begin{array}{l} \mathrm{U}_{\alpha \varepsilon A} M_\alpha \\ \Sigma_{\alpha \varepsilon A} M_\alpha \end{array}\right\}$	or	union of all the sets capital m sub alpha with alpha an element of capital a sum of all the sets capital m sub alpha with alpha an element of capital a	
$\left.\begin{array}{l} \sim M \\ C(M) \\ \bar{M} \\ \tilde{M} \\ M' \end{array}\right\}$		complement of the set capital m	
$\left.\begin{array}{l} M - N \\ M \sim N \end{array}\right\}$	or	complement of capital n in capital m relative complement of capital n in capital m	
$M \sim N$	or	the sets capital m and capital n are bijective the sets capital m and capital n can be put into one to one correspondence	
$M \cap (N \cup P)$		intersection of capital m and the set capital n union capital p	
$M \cap N \cup M \cap P$		capital m intersect capital n union capital m intersect capital p	
$M \cup (N \cap P)$		capital m union the set capital n intersect capital p	
$\overline{(M \cup N)}$		complement of the set capital m union capital n	
$\bar{M} \cap \bar{N}$		intersection of the complement of capital m and the complement of capital n	

Expression	Speak	Notes
$\aleph$	ah́ lef	the first letter of the Hebrew alphabet
$\aleph_0$	ah́ lef null	the cardinal number of the set of positive numbers
$M \simeq N$	capital m and capital n are of the same ordinal type	
ω	omega	the ordinal number of the positive integers in their natural order
ω^*	omega superscript star	the ordinal number of the negative integers in their natural order
${}^*\omega$	left superscript star omega	
π	pi	the ordinal number of all integers in their natural order
Q.E.D.	q e d	"Quod erat demonstrandum" Latin meaning: which was to be demonstrated or which was to be proved

SECTION VII – ELEMNTARY AND ANALYTIC GEOMETRY

Symbol		Speak	Notes
$\angle$		angle	Example: $\angle$ ABC is read the angle ABC.
$\angle$s		angles	
$\perp$		perpendicular	Example: AB$\perp$ CD is read AB is perpendicular to CD.
$\perp$s		perpendiculars	
$\parallel$		parallel	Example: AB $\parallel$ CD is read AB is parallel to CD.
$\parallel$s		parallels	
$\cong$ $\equiv$	or	congruent is congruent to	Example : A$\cong$B is read A is congruent to B.
$\sim$	or	similar is similar to	Example: A $\sim$ B is read A is similar to B.
Δ		triangle	
▱		parallelogram	
□		square	
○		circle	
ⓢ		circles	
π		pi	See Greek alphabet, Section II.
O		origin	
(a,b)		the point a, b	
P(a,b) p: (a,b)		the point capital p with coordinates a and b	
(r,θ)		the point r, theta in polar coordinates	See Greek alphabet, Section II

Symbol		Speak	Notes
(x,y,z)		the point x,y,z in a rectangular coordinate system in space	
(r,θ,z)		the point r, theta, z in a cylindrical coordinate system in space	See Greek alphabet, Section II.
(r,θ,ϕ) $(p,\theta,\phi$	or	the point r, theta, fi rho, theta, fi in a spherical coordinate system in space	See Greek alphabet, Section II.
$\overline{AB}$ AB	or	the line segment a b the line segment between a and b	
$\overrightarrow{AB}$	or	the directed line segment from a to b the ray from a to b	
$\overset{\frown}{AB}$	or	the arc a b the arc between a and b	

SECTION VIII – STATISTICS AND MATHEMATICS OF FINANCE

Greek alphabet-the pronunciation of the Greek letters can be found in Section II.

Symbol		Speak	Notes
χ^2		chi-square	
d.f.		degrees of freedom	
F		capital f	F ratio
i		i	width of a class interval
k		k	coefficient of alienation
P.E.	or	probable error probable deviation	
r	or	r correlation coefficient	Pearson product moment, correlation coefficient between two variables
$r_{12 \bullet 34 \ldots n}$		r sub the quantity one two dot three four dot dot dot n	partial correlation coefficient between variables one and two in a set of n variables
$\left.\begin{matrix} s \\ sd \end{matrix}\right\}$		standard deviation	from a sample
σ_x		sigma sub x	standard deviation of the population of x
σ_{xy}		sigma sub x y	standard error of estimate, standard deviation of an x array for a given value of y
t	or or	t students' t statistic students' t test	
V		capital v	coefficient of variation
$\bar{x}$		x bar	arithmetic average of the variable x from a sample

Symbol	Speak	Notes
μ	mu	arithmetic mean of a population
μ_2	mu sub two	second moment about the mean
μ_r	mu sub r	r^{th} moment about the mean
β_1	beta sub one	coefficient of skewness
β_2	beta sub two	coefficient of kurtosis
$\beta_{12 \bullet 34}$	beta sub the quantity one two dot three four	multiple regression coefficient in terms of standard deviation units
η	eta	correlation ratio
z	z	Fisher's z statistic
Q_1	capital q sub one	first quartile
Q_3	capital q sub three	third quartile
$E(x)$	capital e of x	expected value of x, expectation of x
$P(x_1)$	capital p of x sub i	probability that x assumes the value x sub i
%	percent	
$\$$ or	dollar dollars	
¢ or	cent cents	
@	at	Example: three oranges @ $1.00 each is read three oranges at one dollar each.
$J_{(p)}$	j sub p in parentheses	nominal rate (p conversion periods per year)
n	n	number of periods or years

Symbol	Speak	Notes
l_x	l sub x	number of persons living at age x (mortality table)
d_x	d sub x	number of deaths per year of persons of age x (mortality table)
P_x	p sub x	probability of a person of age x living one year
q_x	q sub x	probability of a person of age x dying within one year
${}_nA_x$	left-subscript n capital a sub x	net single premium for $1 of term insurance for n years for a person aged x
${}_nP_x$	left-subscript n capital p sub x	premiums for a limited payment life policy of $1 with a term of n years at age x
$s_{\overline{n\rceil}}$	s sub n right angle	compound amount of $1 per annum for n years at a given interest rate

SECTION IX – CALCULUS AND ANALYSIS

Greek alphabet – the pronunciation of the Greek letters can be found in Section II.

Expression		Speak	Notes
a		a	usually means acceleration
I		capital i	usually means inertia
k		k	usually means radius of gyration
s, σ		s sigma	usually means length of arc
$\dot{s}$, v		s dot v	usually means velocity
$\overline{x}, \overline{y}, \overline{z}$		x bar, y bar, z bar	
(a, b)	or	open interval a b point a b	
$[a, b]$		closed interval a b	
$(a, b]$	or	interval a less than x less than or equal to b interval a b, open on the left and closed on the right	
$[a, b)$	or	interval a less than or equal to x less than b interval a b, closed on the left and open on the right	
$[x]$	or	greatest integer not greater than x integer part of x	
$\{a_n\}$, $[a_n]$, (a_n)		sequence a sub 1, a sub 2, dot dot dot, a sub n, dot dot dot	
$\sum$		summation	boldface capital sigma

Expression		Speak	Notes
$\sum_{1}^{N}$		summation from one to capital n	
$\sum_{i=1}^{\infty} x_i$		summation from i equals one to infinity of x sub i	
$\prod$		product	boldface capital pi
$\prod_{1}^{n}$		product from one to n	
$\prod_{i=1}^{\infty} y_i$		product from i equals one to infinity of y sub i	
l.u.b.		least upper bound	
sup	or	supremum soup	
g.l.b.		greatest lower bound	
inf	or	inferior inf	
$\left.\begin{array}{l}\lim\limits_{x\to a} y = b \\ \lim\limits_{x=a} y = b\end{array}\right\}$		limit as x approaches a of y equals b	
$\overline{\lim}\, t_n$ $n \to \infty$		limit superior as n approaches infinity of t sub n	
$\underline{\lim}_{n\to\infty}\ t_n$		limit inferior as n approaches infinity of t sub n	

Expression		Speak	Notes
$\limsup$ or $\overline{\lim}$	or	limit superior lim soup	
$\liminf$ or $\underline{\lim}$	or	limit inferior lim inf	
$f(x)$		f of x	
$f(g(x))$ or $f \circ g(x)$	or	f composed with g of x f of g of x	
$f(a+0)$		f of the quantity a plus zero	
$f(a+)$		f of the quantity a plus	
$f(a-0)$		f of the quantity a minus zero	
$f(a-)$		f of the quantity a minus	
$\lim\limits_{x \downarrow a} f(x)$		limit as x decreases to a of f of x	
$\lim\limits_{x \to a+} f(x)$		limit as x approaches a plus of f of x	
$\lim\limits_{x \uparrow a} f(x)$		limit as x increases to a of f of x	
$\lim\limits_{x \to a-} f(x)$		limit as x approaches a minus of f of x	
$f'(a+)$		derivative on the right of f at a	
$f'(a-)$		derivative on the left of f at a	
Δy	or	capital delta y an increment of y	
∂y	or or	partial y a variation in y an increment of y	

Expression		Speak	Notes
dy	or	dy differential of y	
$\frac{dx}{dt}$	or or	derivative with respect to t of x derivative of x with respect to t dx over dt	
$\frac{df(x_0)}{dx}$	or	derivative with respect to x of f at x sub zero derivative of f at x sub zero with respect to x	
y'		y prime	
$f'(x)$		f prime of x	
$D_x\ (y)$	or	derivative with respect to x of y capital d sub x of y	
$\frac{d^n y}{dx^n}$		n^{th} derivative with respect to x of y	
$y^{(n)}$		y to the n^{th} prime	
$\left.\begin{matrix} p' \\ \dot{p} \end{matrix}\right\}$	or	p prime first derivative of p	
$\left.\begin{matrix} p'' \\ \ddot{p} \end{matrix}\right\}$	or	p double prime second derivative of p	
$f^{(n)}(x)$		f to the n^{th} prime of x	
$D_x^n y$	or	n^{th} derivative with respect to x of y capital d sub x super n of y	
$f'(g(x))$	or	f prime of g of x f prime at g of x	
$(f(g(x)))'$		the quantity f of g of x, that quantity prime	
$f'(g(x))g'(x)$		the product of f prime of g of x and g prime of x	

Expression		Speak	Notes
$(f(x)g(x))'$		the quantity f of x times g of x, that quantity prime	
$f'(x)g(x)+f(x)g'(x)$		f prime of x times g of x, that product plus f of x times g prime of x	
$\left(\dfrac{f(x)}{g(x)}\right)'$		the quantity f of x over g of x, that quantity prime	
$\dfrac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$		the fraction, the numerator is f prime of x times g of x, that product minus f of x times g prime of x, the denominator is g squared of x	
$f(x,y)$		f of x, y	
$\dfrac{\partial u}{\partial x}$		partial derivative of u with respect to x	
	or	partial u over partial x	
	or	partial derivative with respect to x of u	
u_x		partial derivative of u with respect to x	
	or	u sub x	
$f_x(x,y)$		partial derivative with respect to x of f of x, y	
	or	f sub x of x, y	
$f_1(x,y)$		partial derivative with respect to the first variable of f of x, y	
	or	f sub one of x, y	
$\dfrac{\partial^2 u}{\partial y \partial x}$		second partial derivative of u, first with respect to x and then with respect to y	
u_{xy}		second partial derivative of u, first with respect to x and then with respect to y	
	or	u sub x y	

Expression		Speak	Notes
$f_{xy}(x,y)$	or	second partial derivative of f of x,y, first with respect to x and then with respect to y f sub x y of x, y	
$f_{12}(x,y)$		second partial derivative of f of x, y, first with respect to the first variable and then with respect to the second variable f sub one two of x, y	
$D_y(D_x u)$	or	partial derivative with respect to y of the partial derivative with respect to x of u capital d sub y of capital d sub x u	
D		operator d over d x capital d	
$f(x_1, x_{2,\ldots,} x_n)$		f of x sub one, x sub two, dot dot dot, x sub n	
D_i	or	partial derivative with respect to the i[th] variable capital d sub i	Example: $D_i f(x_1, x_2, \ldots, x_n) = \frac{\partial}{\partial x_i} f(x_1, x_{2,}, \ldots, x_n)$
D_{ij}	or	second partial derivative first with respect to x sub i then with respect to x sub j capital d sub i j	Example: $D_{ij} f(x_1, x_2, \ldots, x_n) = D_i(D_j f(x_1, x_2, \ldots, x_n))$
$D_s f$	or	directional derivative of f in the direction s capital d sub s of f	
$\Delta f(x)$		delta f of x	
∇		del	
∇f	or	gradient of f del f	
$\vec{u}$		vector u	

Expression		Speak	Notes
grad f		gradient of f	
$\nabla \cdot \vec{u}$	or	divergence of vector u del dot vector u	
div $\vec{u}$	or	divergence of vector u div vector u	
$\nabla \times \mathbf{F}$	or	curl of boldface capital f del cross boldface capital f	
∇^2	or	Laplacian operator del squared	Example: $\nabla^2 u$ is the Laplacian operator on u.
Δ	or	Laplacian operator delta	
δ^i_j		Kronecker delta	
$F(x)\Big]^b_a$	or	capital f of x evaluated from a to b capital f of b minus capital f of a	
$\int f(x)dx$	or	integral f of x d x anti derivative of f with respect to x	
$\int_a^b f(x)dx$		integral from a to b of f of x d x	
$\overline{\int_a^b}$		upper Darboux integral from a to b	
$\underline{\int_a^b}$		lower Darboux integral from a to b	
$\int_a^b \left[\int_c^d f(x,y)dy \right] dx$		iterated integral: integral from a to b of the integral from c to d of f of x, y d y d x	
$\int_\alpha^\beta \left[\int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta)rdr \right] d\theta$		iterated integral: integral from alpha to beta of the integral from r sub one of theta to r sub two of theta of f of r, theta r d r d theta	iterated integral in polar coordinates

Expression	Speak	Notes
$\int_R 1\ dV$	integral over capital r of one d capital v	
$\int_0^{2\pi}\left(\int_0^a\left(\int_{-\sqrt{a^2-r^2}}^{\sqrt{a^2-r^2}} 1\cdot r\ dz\right)dr\right)d\theta$	iterated integral: integral from zero to two pi of the integral from zero to a of the integral from minus square root of the quantity a squared minus r squared to square root of the quantity a squared minus r squared of one dot r d z d r d theta	iterated integral with cylindrical coordinates
$\int_0^{2\pi}\left(\int_0^{\pi/2}\left(\int_0^a \rho\cos\phi\rho^2 \sin\ \phi\ d\ \rho\right)d\ \phi\right)d\theta$	iterated integral: integral from zero to two pi of the integral from zero to pi over two of the integral from zero to a of rho cosine fi rho squared sine fi d rho d fi d theta	iterated integral with spherical coordinates
$\int_R f\ d\ V$	integral over capital r of f d capital v	
$\int f[x(u)]\frac{dx}{du}du$	integral of the product of three factors: f of x of u, and d x over d u, and d u or integral of f of x of u times d x over d u times d u	
$\int_{a_1}^{b_1}\int_{a_2}^{b_2}\cdots\int_{a_n}^{b_n} f(x_1,x_2,\ldots x_n)dx_1dx_2\ldots dx_n$	multiple integral: integral from a sub one to b sub one, integral from a sub two to b sub two, dot dot dot, integral from a sub n to b sub n of function f of x sub one, x sub two, dot dot dot, x sub n, end of function, d x sub one d x sub two dot dot dot d x sub n	
$\int_\gamma f(z)dz$	integral over gamma of f of z d z	
$\oint_C M(x,y)dx$	line integral along capitol c in positive direction of function capital m of x, y d x	

Expression	Speak	Notes
$\iint\limits_S g(x,y,z)dS$	surface integral over capital s of g of x, y, z d capital s	
$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(t)e^{ixt}dt$	one divided by the square root of two pi that fraction times integral from minus infinity to infinity of the quantity g of t times e to the i x t power d t	
(f,g)	inner product of the functions f and g	
$\lVert f \rVert$	norm of the function f	
$f * g$	convolution of f and g	
$W(u_1 u_2,\ldots,u_n)$	Wronskian of u sub one, u sub two, dot dot dot u sub n	
$\left.\begin{array}{l}\frac{\partial(f_1,f_2,\ldots f_n)}{\partial(x_1,x_2,\ldots,x_n)} \\ \frac{D(f_1,f_2,\ldots,f_n)}{D(x_1,x_2,\ldots,x_n)} \\ J\left(\frac{f_1,f_2,\ldots,f_n}{x_1,x_2,\ldots,x_n}\right)\end{array}\right\}$	Jacobian of the function f sub one of x sub one, x sub two, dot dot dot x sub n; f sub two of x sub one, x sub two, dot dot dot, x sub n; dot dot dot f sub n of x sub one, x sub two, dot dot dot x sub n	

In the following expressions z is a complex number.

$\lvert z \rvert$	or	absolute value of z modulus of z
$\bar{z}$	or	conjugate of z z bar
conj z		conjugate of z
arg z		argument of z

Expression	Speak	Notes
$\left.\begin{array}{l} \mathrm{R}(z) \\ \mathcal{R}(z) \\ \mathrm{Re}(z) \end{array}\right\}$	real part of z	
$\left.\begin{array}{l} \mathrm{I}(z) \\ \mathcal{I}(z) \\ \mathrm{Im}(z) \end{array}\right\}$	imaginary part of z	
$\operatorname*{Res}_{z=a} f(z)$	residue at z equals a of f of z	

SECTION X – LINEAR ALGEBRA

Note: Matrices are read either by rows or by columns and the number of rows and columns determines the size of the matrix. Hence, a matrix with four rows and three columns is called a four-by-three matrix. (The number of rows is listed first, i.e., 4 by 3.)

Expression		Speak	Notes
$\begin{bmatrix} 2, & 7 \\ 3, & 10 \end{bmatrix}$		two by two matrix first row two seven second row three ten	
$\begin{bmatrix} 2 & 7 \\ 3 & 10 \end{bmatrix}$	or	two by two matrix first column two three second column seven ten	
a_{ij}		a sub i j	
m x n	or	m cross n m by n	
$a_{i+1,j}$		a double subscript i plus one comma j	
$a_{i,j-1}$		a double subscript i comma j minus one	
$a_{i+\frac{1}{2},j-\frac{1}{2}}$		a double subscript i plus one half, j minus one half	
$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$		m by n matrix: first row a sub one one, a sub one two, dot dot dot, a sub one n second row a sub two one, a sub two two, dot dot dot, a sub two n third row dot dot dot m^{th} (or last) row a sub m one, a sub m two, dot dot dot a sub m n	

Expression		Speak	Notes
$[a_{ij}]1 \le i \le m, 1 \le j \le n$		the m by n matrix with elements a sub i j	
	or	open bracket a sub i j close bracket one less than or equal to i less than or equal to m, one less than or equal to j less than or equal to n	
$a\mathbf{A}$		a boldface capital a	
	or	scalar product of a and matrix boldface capital a	
$\mathbf{A}^{T}$		boldface capital a superscript capital t	
	or	transpose of the matrix boldface capital a	
	or	the matrix boldface capital a transpose	
$\mathbf{A}'$		boldface capital a prime	
	or	transpose of the matrix boldface capital a	
	or	the matrix boldface capital a transpose	
$\mathbf{A}^{H}$		boldface capital a superscript capital h	
	or	Hermitian transpose of the matrix boldface capital a	
$[\mathbf{AB}]^{-1}$		left bracket boldface capital a boldface capital b right bracket superscript minus one	
	or	inverse of the matrix product boldface capital a boldface capital b	
$[\mathbf{A}+\mathbf{B}]^{-1}$		left bracket boldface capital a plus boldface capital b right bracket superscript minus one	
	or	inverse of the matrix sum boldface capital a plus boldface capital b	
$\mathbf{ABB}^{-1}$		boldface capital a boldface capital b boldface capital b superscript minus one	
	or	the product boldface capital a boldface capital b boldface capital b inverse	

Expression		Speak	Notes
$\mathbf{A}^{-1}$	 or or	boldface capital a superscript minus one inverse of the matrix boldface capital a matrix boldface capital a inverse	
$\|\mathbf{A}\|$ $\det\ \mathbf{A}$		determinant of the square matrix boldface capital a	
$\det\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & \cdots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$		determinant of the matrix: first row a sub one one, a sub one two, dot dot dot, a sub one n second row a sub two one, a sub two two, dot dot dot, a sub two n third row dot dot dot n^{th} (or last) row a sub n one, a sub n two, dot dot dot, a sub n n	
$\sum_{k=1}^{n} a_{ik}b_{kj}$		summation from k equals one to n of the product a sub i k b sub k j	
$\sum_{s=1}^{n}\sum_{t=1}^{p} a_{is}b_{st}c_{tj}$		summation from s equals one to n of summation from t equals one to p of the product a sub i s b sub s t and c sub t j	
$ax + by = e$ $cx + dy = f$		the system of equations first equation: a x plus b y equals e second equation: c x plus d y equals f	

Expression	Speak	Notes
$a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n = b_1$ $\ldots$ $a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n = b_2$ $\ldots$ $a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n = b_m$	the system of equations first equation: a sub one one x sub one plus a sub one two x sub two plus dot dot dot plus a sub one n x sub n equals b sub one second equation: a sub two one x sub one plus a sub two two x sub two plus dot dot dot plus a sub two n x sub n equals b sub two third line: dot dot dot m^{th} (or last) equation: a sub m one x sub one plus a sub m two x sub two plus dot dot dot plus a sub m n x sub n equals b sub m	
$[a_1, a_2, \ldots\ a_n]$	n row vector a sub one a sub two dot dot dot a sub n	
$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$	n column vector a sub one a sub two dot dot dot a sub n	
$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & 1 \end{bmatrix}$	matrix: first row one zero dot dot dot zero second row zero one dot dot dot zero third row dot dot dot last row zero zero dot dot dot one or identity matrix	
I	identity matrix	

Expression	Speak	Notes
$\begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & \cdots & d_n \end{bmatrix}$	matrix: first row d sub one zero zero dot dot dot zero second row zero d sub two zero dot dot dot zero third row dot dot dot n^{th} (or last) row zero zero zero dot dot dot d sub n	
or	n by n diagonal matrix with d sub one to d sub n on the diagonal	
$\begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ 0 & u_{22} & \cdots & u_{2n} \\ & & \cdots & \\ 0 & \cdots & 0 & u_{nn} \end{bmatrix}$	matrix: first row u sub one one u sub one two dot dot dot u sub one n second row zero u sub two two dot dot dot u sub two n third row dot dot dot n^{th} (or last) row zero dot dot dot zero u sub n n	
or	n by n upper triangular matrix	

Expression	Speak	Notes
$\begin{bmatrix} \ell_{11} & 0 & \cdots & 0 \\ \ell_{21} & \ell_{22} & \cdots & 0 \\ & & \cdots & \\ \ell_{n1} & \ell_{n2} & \cdots & \ell_{nn} \end{bmatrix}$	matrix: first row script l sub one one zero dot dot dot zero second row script l sub two one script l sub two two dot dot dot zero third row dot dot dot n^{th} (or last) row script l sub n one script l sub n two dot dot dot script l sub n n	
or	n by n lower triangular matrix	
$\begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots \\ \ell_{21}u_{11} & \ell_{21}u_{12} + u_{22} & \ell_{21}u_{13} + u_{23} & \cdots \\ \ell_{31}u_{11} & \ell_{31}u_{12} + \ell_{32}u_{22} & \ell_{31}u_{13} + \ell_{32}u_{23} + u_{33} & \cdots \\ & \cdots & & \end{bmatrix}$	matrix: first row first element u sub one one second element u sub one two third element u sub one three, dot dot dot second row first element script l sub two one u sub one one second element script l sub two one u sub one two plus u sub two two third element script l sub two one u sub one three plus u sub two three, dot dot dot third row first element script l sub three one u sub one one second element script l sub three one u sub one two plus script l sub three two u sub two two third element script l sub three one u sub one three plus script l sub three two u sub two three plus u sub three three, dot dot dot fourth row dot dot dot	

SECTION XI – TOPOLOGY AND ABSTRACT SPACES

Note: In the following expressions, the capital letters M and N denote sets.

Expression	Speak	Notes
$\overline{M}$	capital m bar	closure of capital m
M'	capital m prime	derived set of capital m
$d(x,y)$ $\delta(x,y)$ $\rho(x,y)$ (x,y)	d of x,y delta of x,y rho of x,y x,y	distance from x to y
$M \times N$	capital m cross capital n	the Cartesian product of spaces capital m and capital n
M/N	capital m slash capital n	the quotient space of capital m and capital n
E_n E^n R_n R^n	capital e sub n capital e superscript n capital r sub n capital r superscript n	real n-dimensional Euclidean space
Z_n C_n	capital z sub n capital c sub n	complex n-dimensional space
H $\mathfrak{H}$	capital h Gothic capital h	Hilbert space
$(\mathbf{x},\mathbf{y})$	open parenthesis boldface x, boldface y closed parenthesis	inner product of the elements x and y of a vector space
$\|\mathbf{x}\|$	norm of boldface x	
l_p $l^{(p)}$	italic l sub p space italic l superscript p in parentheses space	

Expression	Speak	Notes
L_p	capital l sub p space	
$L^{(p)}$	capital l superscript p in parentheses space	
$\left[\sum_{i=1}^{\infty} \lvert x_i \rvert^p\right]^{1/p}$	summation i equals one to infinity of the absolute value x sub i, that absolute value raised to the p power, and the whole sum raised to the one over p power	
$\left[\int_s \lvert f(x) \rvert^p ax\right]^{1/p}$	integral over s of the absolute value of f of x, that absolute value raised to the p power a x and the whole integral raised to the one over p power	
∂S	partial capital s	boundary of the set capital s
ΔS	capital delta capital s	
$d(S)$	d of capital s	

Section XII Diagrams and Graphs

In this section the approach changes from previous sections. Here suggestions are merely offered to alleviate the very complicated problem of diagram description.

Diagrams are visual aids and are very useful to illustrate qualitative information. Because of their visual nature, it is somewhat clumsy and sometimes even impossible to describe them verbally. The old saying, “A picture is worth a thousand words”, sums up the difficulty faced when trying to describe a picture with words. The degree of complexity of the diagram should determine whether “reading” the diagram is worth the effort. Some illustrations require so many words from the reader that it can render the listener in a state of depressed confusion from which there is no reasonable hope of bringing him out clear-headed again.

This section deals mainly with suggestions for describing diagrams in general. These suggestions should help the interpreter convey the information in the illustration to the listener in as clear a manner as possible. It is most important that diagrams be described clearly. A poorly read diagram is worse than one not read at all, because it can confuse and frustrate the listener and even give misleading information. When taping, if the reader finds that the material to be described is not clear or comprehensive to himself, the reader should consult the listener in person. Specific questions from the listener will likely elicit the desired information. If the listener is blind, there are other ways to facilitate understanding of the diagram, such as tracing the diagram using the blind person’s hand, or using raised line drawing paper to duplicate the essential parts.

The following are some specific suggestions that I have personally found helpful when having diagrams read to me. First, read the caption, for it may contain a very good description of the diagram itself. Next, describe the shapes either contained in the diagram or comprising the entire diagram. An example of the former case is a flow chart, a chart consisting of circles, squares, triangle, etc., with connecting arrows. An example of the latter case would be a pie diagram, where a circle is cut into pie-shaped sections or wedges. Besides stating the basic geometric shapes, use words for the shapes of any familiar objects, such as crescent, football, piece of bread, sausage, tear drop, etc. Describe the orientation of the various figures in the diagram, i.e., how the various figures are related to one another. Describe the basic layout, if there is one.

An important subcategory of the diagram is the graph. Particularly in mathematics, graphs are widely used. Often they are hard to describe, for they can depict complicated figures, such as the projection of a three-dimensional object on a plane. Nonetheless, from my experience, having certain key features of a graph described facilitates the listener’s understanding of whatever the graph is depicting.

First, a framework upon which the graph is constructed is needed. In a graph, the horizontal and vertical lines form the axes of a coordinate system. The horizontal line in general is known as the x-axis and the vertical line as the y-axis. (Any letters may be used to label the axes.) If there is a scale marked on the axes, for the horizontal axis it increases

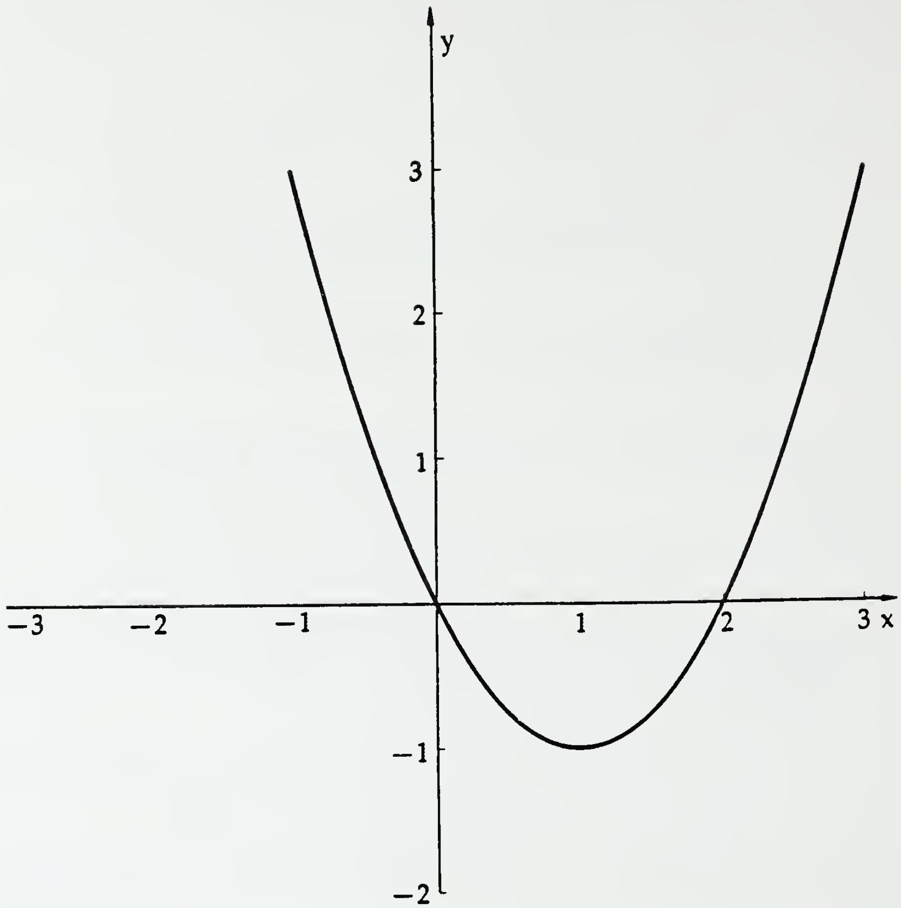
from left to right; for the vertical axis it increases from down to up. The point where the axes meet is the origin. The axes divide the plane into four quadrants: the upper right is the first, upper left is the second, lower left is the third, and lower right is the fourth. This is the basic framework upon which the graph is constructed.

The following is a list of some of the key features of a graph that should be described:

- Read the labels on the axes and any marking or scale on the axes.
- If possible, read from left to right, and state in which quadrant the graph begins and in which it ends.
- As the graph traverses from left to right, state where it goes up or down and over what point on the x-axis it changes direction.
- Describe how steeply each portion of the graph goes up or down. Compare that portion to a line which forms a particular angle with the x-axis, such as 15°, 30°, 45°, etc., if desired.
- State at what points the graph crosses the axes, and where it reaches its local minima or maxima.
- Describe the shapes of the various portions of the graph. Examples of shapes are: straight line, semicircle, parabola, sinusoid, etc.
- Describe the concavity of the various portions of the graph; specify which portion is concave up (a curve that opens up or a dip) and which portion of the graph is concave down or convex (a curve that opens down or a hump).
- Describe the point of inflection, i.e., the point on the graph at which the graph changes concavity.
- Specify any points of discontinuity (breaks in the graph) and any cusps (sharp points on the graph).
- Describe the symmetry of the graph (i.e., on which line one half of the graph is the mirror image of the other.
- If there is more than one graph in the figure, describe each graph individually, and describe where they intersect or how they are related to each other.

The types of diagrams and graphs are so varied that these few pages cannot help specifically in every case. These suggestions are limited, but it is hoped that not only will they be useful in themselves, but also will inspire the interpreter to develop his own ideas to describe diagrams clearly.

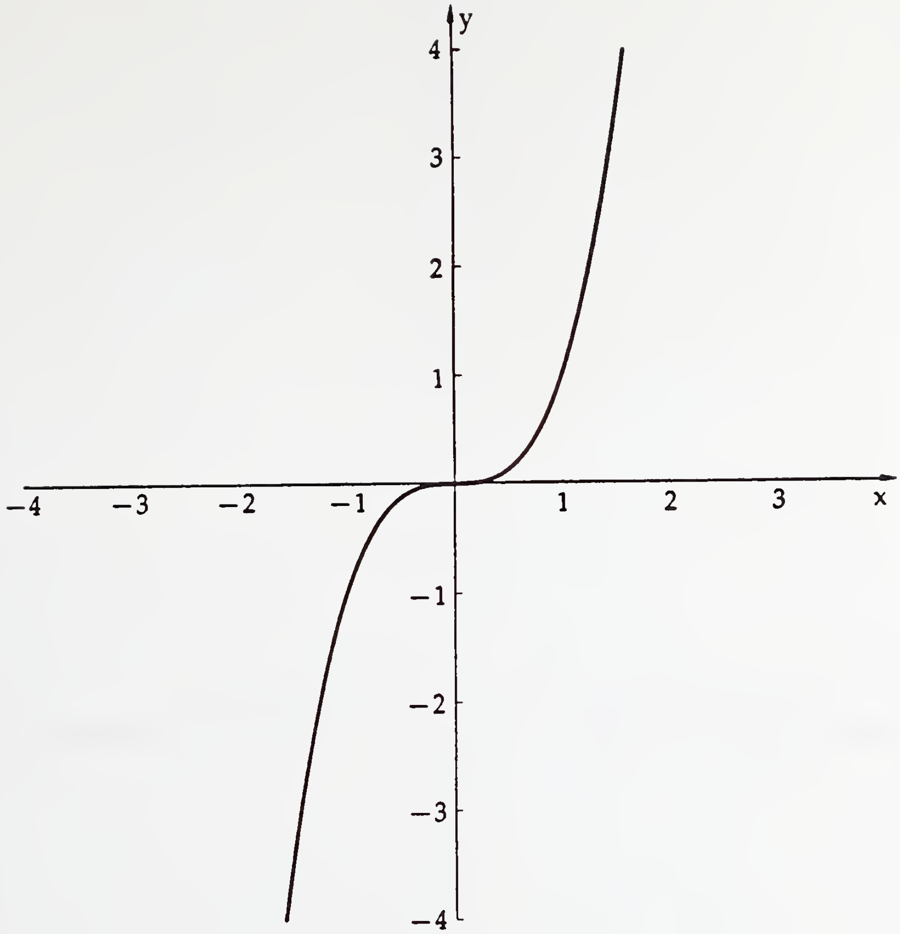
This section concludes with a few examples of graphs, each (except the last) accompanied by a suggested verbal description. The last one cannot be reasonably described.



$y = x^2 - 2x$, a parabola

Speak:

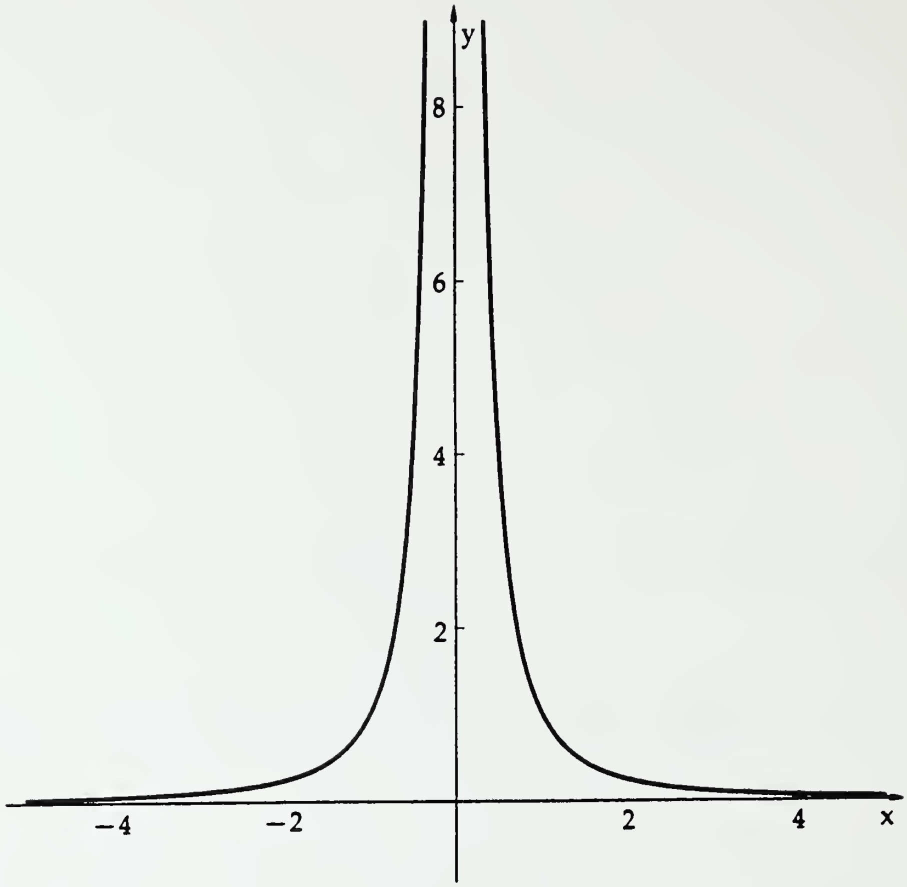
The graph is captioned: y equals x squared minus two x, a parabola. The graph has x- and y-axes and the scale for both axes is in units of one, labeled from minus three to plus three. The shape of the graph is a parabola, concave up. It is symmetric about the vertical line x equals one. The graph begins in the second quadrant and decreases steeply, almost vertically, from the upper left as it moves to the right. It crosses the origin and continues to go down into the fourth quadrant and reaches the minimum at the point one, minus one. The graph then changes direction to go up and crosses the x-axis again at the point two, zero, moves into the first quadrant and continues to go up steeply.



$y = x^3$, the cubic

Speak:

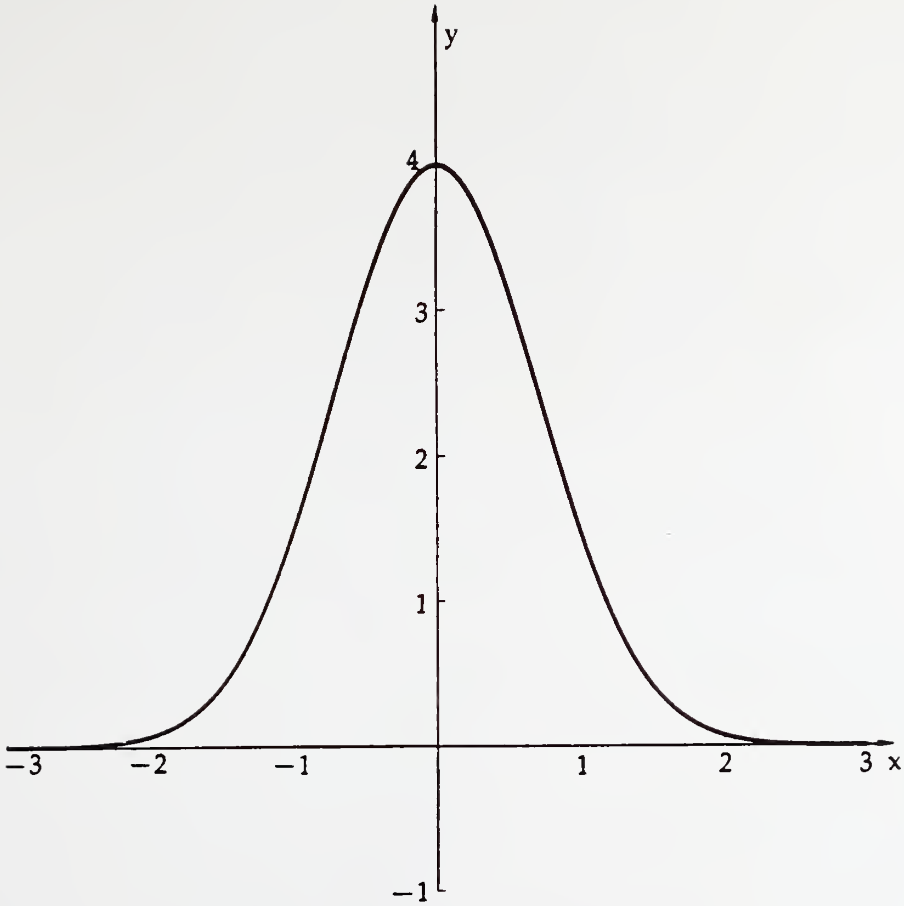
The graph is captioned: y equals x cubed, the cubic. The graph has x- and y-axes, and the scale for both axes is in units of one, labeled from minus four to plus four. The graph is antisymmetric about the vertical line x equals zero, the y-axis. The graph begins in the third quadrant and increases steeply as it moves to the right. As it nears the origin it flattens out somewhat, crosses the axes at the origin, remains somewhat flat close to the origin, after which it increases steeply again in the first quadrant. It is concave down for x less than zero and concave up for x greater than zero.



$$y = \frac{1}{x^2}$$

Speak:

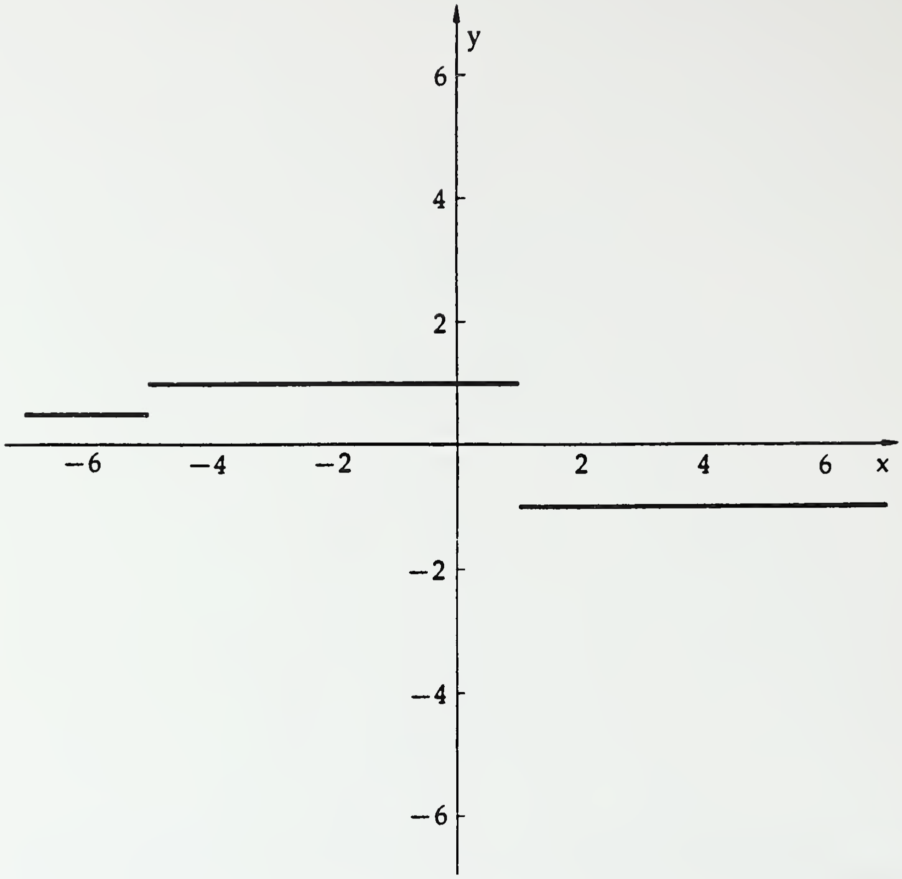
The graph is captioned: y equals the fraction one over x squared. The graph has x- and y-axes and the scale for both axes is in units of two, labeled from minus four to plus eight. The graph is symmetric about the vertical line x equals zero, the y-axis. The graph consists of two separate branches. The first begins in the second quadrant very close to the x-axis. As it moves to the right, the graph increases very slowly until it reaches the point minus one, one. As it continues to approach zero from the left, the graph increases steeply and nears but never touches the y-axis. That is the end of the first branch of the graph, which is entirely contained in the second quadrant. The graph has a discontinuity at x equals zero. The second branch of the graph is entirely contained in the first quadrant. It begins very close to the y-axis. As it moves to the right, the graph decreases steeply, until it reaches the point one, one, where it begins to flatten out, and slowly approaches the x-axis but never touches it. That is the end of the second branch of the graph.



$y = 4e^{-x^2}$

Speak:

The graph is captioned: y equals four times e raised to the quantity minus x squared. The graph has x- and y-axes and the scale for both axes is in units of one, labeled from minus three to plus four. The graph is a bell-shaped curve symmetric about the y-axis and concave down. The graph begins in the second quadrant near the x-axis. When x is less than minus two, the graph increases slowly. When x is greater than minus two and less than zero, the graph increases sharply and crosses the y axis at the point zero, four. The graph then decreases rapidly for x greater than zero and less than two. For x greater than two, it decreases slowly as it approaches the x-axis but never touches it.



$$\text{The step function: } y = \begin{cases} \frac{1}{2}, & x < -5 \\ 1, & -5 \leq x < 1 \\ -1, & x \geq 1 \end{cases}$$

Note: This graph is an example where the caption obstensively describes the graph.

Speak:

The graph is captioned: the step function: y equals one half when x is less than minus five, y equals one, when minus five is less than or equal to x is less than one, and y equals minus one, when x is greater than or equal to one. The graph has x- and y-axes and the scale for both axes is in units of two, labeled from minus six to plus six. The graph consists of three disjoint horizontal line segments parallel to the x-axis. The first line segment is located at y equals one half when x is less than minus five. It is entirely contained in the second quadrant. The second line segment is located at y equals one for x greater than or equal to minus five and less than one. It begins in the second quadrant, crosses the y-axis and ends near the point x equals one in the first quadrant. The third line segment is located at y equals minus one when x is greater than one. It is entirely contained in the fourth quadrant.

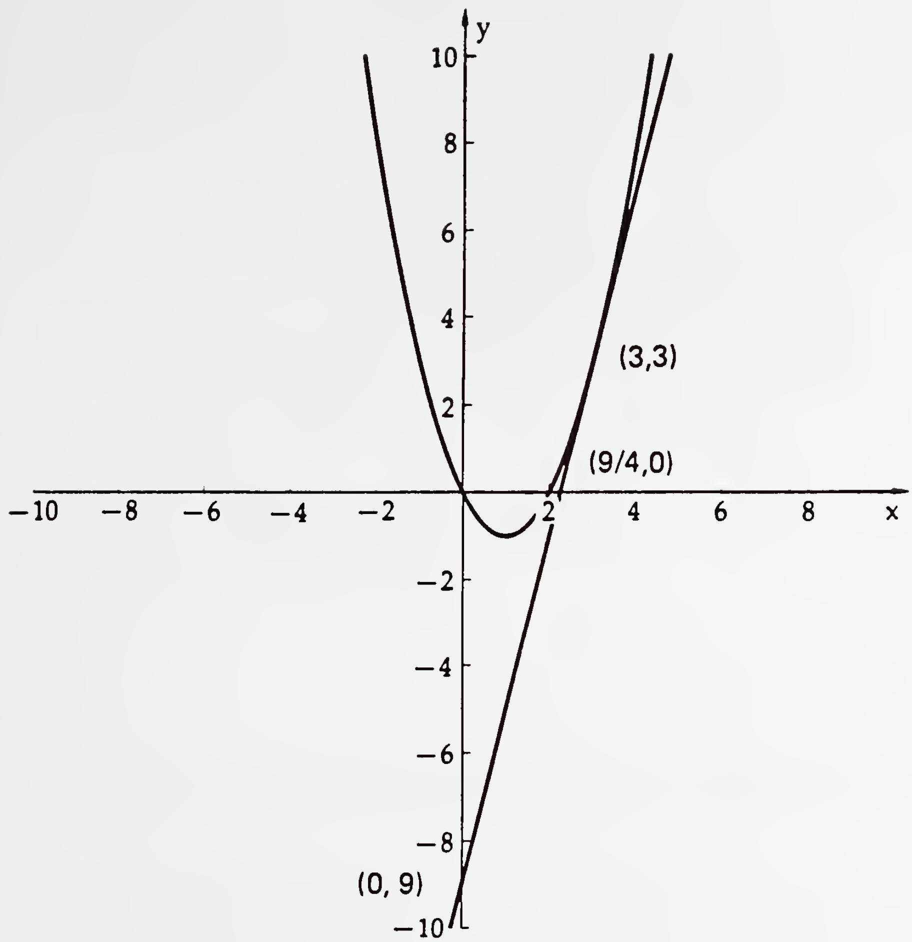
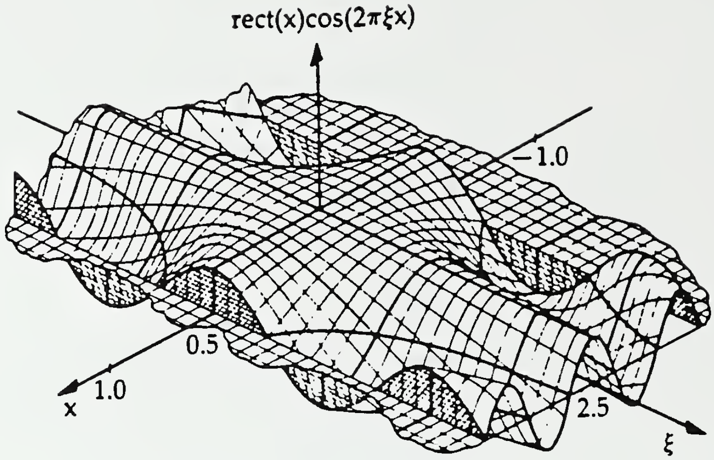


Diagram containing 2 graphs: $y = x^2 - 2x$ and $y = 4x - 9$

Speak:

The graph is captioned: diagram containing two graphs: y equals x squared minus two x and y equals four x minus nine. The scale for the x- and y-axes is in units of two and is labeled from minus ten to plus ten. The parabola is described as in Example 1. The second graph is a straight line which starts in the third quadrant, intersects the y-axis at the point zero, minus nine, and continues through the fourth quadrant. It intersects the x-axis at the point nine fourths, zero, and continues up into the first quadrant. The angle between the graph and the x-axis is fairly close to ninety degrees. The two graphs, the parabola and the straight line intersect at the point three, three; or the straight line is tangent to the parabola at the point three, three.



Two-dimensional representation of the integrand of the Fourier integral of the rectangular function of x.

Speak:

The diagram is captioned: two-dimensional representation of the integrand of the Fourier integral of the rectangular function of x.

Comments:

This picture is worth more than a thousand words. This diagram is so complicated that one should probably not consider describing it verbally other than reading the caption. Use of raised line drawing paper or a discussion between reader and listener of some of its main points could be useful.

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APPENDIX C

NEMETH CODE QUICK REFERENCE SHEETS

- Charts and Summary Cards from the Computerized Braille Nemeth Code Tutor (Cortesi)
- Quick Reference Sheets (Buntrock)
- Table of Contents for Modules in the Computerized Braille Nemeth Code Tutor

THE NUMBERS

Note: in horizontal displays, unless preceded by another mathematical symbol, the first braille number of a braille line, or the the first after a space, should have the numeric indicator before it to indicate that what follows is a number.

BRAILLE NEMETH NUMBERS						
(#)	1	2	3	4	5	6
3-4-5-6	dot 2	dots 2-3	dots 2-5	dots 2-5-6	dots 2-6	dots 2-3-5
⠼	⠂	⠆	⠒	⠲	⠢	⠖
numeric indicator	one	two	three	four	five	six

7	8	9	0	EXAMPLES	
dots 2-3-5-6	dots 2-3-6	dots 3-5	dots 3-5-6	10	945
⠶	⠦	⠔	⠴	⠼⠂⠴	⠼⠔⠲⠢
seven	eight	nine	zero	ten	nine hundred forty-five

SIGNS OF OPERATION

+	−	×	·	÷	/	—
3-4-6	3-6	4, 1-6	1-6	4-6, 3-4	4-5-6, 3-4	3-4
⠬	⠤	⠈⠡	⠡	⠨⠌	⠸⠌	⠌
plus (positive)	minus (negative)	multipli- cation cross	multipli- cation dot	divided by	division, slash	division, fraction bar

$\overline{)}$	$\sqrt{}$	±	!	∩	∪	*
1-3-5, 2-5*	3-4-5, 1-2-4-5-6	3-4-6, 3-6	1-2-3-4-6	4-6, 1-4-6	4-6, 3-4-6	4, 3-4-5-6
⠒⠒⠒ ⠕	⠜⠻	⠬⠤	⠯	⠨⠩	⠨⠬	⠈⠼
division, long	radicand (root)**	plus and minus	factorial	intersection	union	asterisk

* row of dots 2-5 in the braille line above the symbol in spatial displays; in horizontal displays, only the curved division symbol (dots 1-3-5) is needed.

** if only the radicand is displayed in print, without the viniculum, then only dots 3-4-5 are to be brailled; in all other cases, the radical needs the termination indicator (dots 1-2-4-5-6). In spatial displays, a row of dots 2-5 is placed above the radicand symbol and extends to the same cell as the termination symbol.

SIGNS OF COMPARISON

For rules on signs of comparison, see card on page 1. In general, space before and after.

SIGNS OF COMPARISON						
$=$	$>$	$<$	$\geq$	$\leq$	$\sim$	$\cong$
4-6, 1-3	4-6, 2	5, 1-3	4-6, 2, 1-5-6	5, 1-3, 1-5-6	4, 1-5-6,	4, 1-5-6, 4-6, 1-3
⠨⠅	⠨⠂	⠐⠅	⠨⠂⠱	⠐⠅⠱	⠈⠱	⠈⠱⠨⠅
equals sign	greater than	less than	greater than or equal to	less than or equal to	similar	congruent

$\neq$	$\ngtr$	$\nless$	$\ngeq$	$\nleq$	$\nsim$	$\ncong$
3-4, 4-6 1-3	3-4, 4-6, 2	3-4, 5, 1-3	3-4, 4-6, 2, 1-5-6	3-4, 5, 1-3, 1-5-6	3-4, 1-5-6	3-4, 1-5-6, 4-6, 1-3
⠌⠨⠅	⠌⠨⠂	⠌⠐⠅	⠌⠨⠂⠱	⠌⠐⠅⠱	⠌⠱	⠌⠱⠨⠅
not equal	not greater	not lesser	not greater than or equal	not less than or equal	not similar to	not congruent

$\equiv$	$\not\equiv$	$\approx$	$\in$	$::$	$:$
4-5-6, 1-2-3	3-4, 4-5-6, 1-3	4, 1-5-6 4, 1-5-6	4, "e"	5-6, 2-3	5, 2
⠸⠇	⠌⠸⠇	⠈⠱⠈⠱	⠈⠑	⠰⠆	⠐⠂
identity	not identity	approx. equals	element of	proportion	ratio (colon)

SIGNS OF COMPARISON (continued)					
$\subset$	$\supset$	$\subseteq$	$\supseteq$	$\rightarrow$	$\leftarrow$
4-5-6, 5, 1-3	4-5-6, 4-6, 2	4-5-6, 5, 1-3, 1-5-6	4-5-6, 4-6, 2, 1-5-6	1-2-4-6, 1-3-5	1-2-4-6, 2-4-6, 2-5, 2-5
⠸⠐⠅	⠸⠨⠂	⠸⠐⠅⠱	⠸⠨⠂⠱	⠫⠕	⠫⠪⠒⠒
proper subset	proper superset	reflex. subset	reflex. superset	left arrow	right arrow

$\perp$	$\parallel$	$\mid$
1-2-4-6, "p"	1-2-4-6, "l"	1-2-5-6
⠫⠏	⠫⠇	⠳
perpendicular to	parallel to	such that

For rules on signs of comparison, refer to Lesson 2 and Lesson 11. In general, space before and after a sign of comparison.

SIGNS OF GROUPING					
(	)	[	]	{	}
1-2-3-5-6	2-3-4-5-6	4, 1-2-3-5-6	4, 2-3-4-5-6	4-6, 1-2-3-5-6	4-6, 2-3-4-5-6
⠷	⠾	⠈⠷	⠈⠾	⠨⠷	⠨⠾
left paranthesis	right paranthesis	left bracket	right bracket	left brace	right brace

SPECIAL SIGNS UNIQUE TO BRAILLE (no print equivalents)					
1-4-5-6	3-4-5-6	4-5-6, 1-4-5-6	4-5-6, 3-4-5-6	1-2-4-5-6	4-5-6
⠹	⠼	⠸⠹	⠸⠼	⠻	⠸
open fraction indicator	close fraction indicator	open mixed number indicator	close mixed number indicator	termination indicator	punctuation indicator

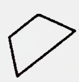
SPECIAL SIGNS UNIQUE TO BRAILLE (no print equivalents)					
4-5	5-6	5	4-6	4-5-6	5-6
⠘	⠰	⠐	⠨	⠸	⠰
superscript indicator	subscript indicator	baseline indicator	italic font	boldface font	English letter indicator

SYMBOLS						
¢	$	%	,	.	°	π
4, “c”	4, “s”	4, 3-5-6	6	4-6	4-6, 1-6	4-6, “p”
⠈⠉	⠈⠎	⠈⠴	⠠	⠨	⠨⠡	⠨⠏
cent sign	dollar sign	percent	comma, mathematical	decimal point	degree*	pi

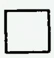
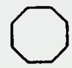
* when the degree symbol is presented with other symbols (e.g., 212°), it is preceded by the superscript indicator (dots 4-5) to indicate its raised position.

∅	∞	∫	∂	∘	|	|
4-5-6, 3-5-6	6, 1-2-3-4-5-6	2-3-4-6	4, “d”	4-6, 1-6	1-2-5-6	6, 1-2-5-6
⠸⠴	⠠⠿	⠮	⠈⠙	⠨⠡	⠳	⠠⠳
empty set	infinity	integral	partial derivative	composite functions	absolute value	matrix/ determ.

SUMMARY OF COMMON SHAPE SYMBOLS

1-2-4-6, "a"	1-2-4-6, "b"	1-2-4-6, "c"	1-2-4-6, "d"	1-2-4-6, "e"	1-2-4-6, "f"	1-2-4-6, "g"
⠫⠁		⠫⠉	⠫⠙	⠫⠑		⠫⠛
⌒		○	◇	⬭		▱
Arc (upward)		Circle	Diamond	Ellipse (oval)		paral-leloGram
1-2-4-6, "h"	1-2-4-6, "i"	1-2-4-6, "j"	1-2-4-6, "k"	1-2-4-6, "l"	1-2-4-6, "m"	1-2-4-6, "n"
⠫⠓	⠫⠊			⠫⠇		
▱	×			=		
rHombus	Intersect-ing lines			paraLLeL to		
1-2-4-6, "o"	1-2-4-6, "p"	1-2-4-6, "q"	1-2-4-6, "r"	1-2-4-6, "s"	1-2-4-6, "t"	1-2-4-6, "u"
⠫⠕	⠫⠏	⠫⠟	⠫⠗	⠫⠎	⠫⠞	
→	⊥		▭	☆	△	
right arrow	Perpen-dicular	Quadri-lateral	Rectangle	Star	Triangle	
1-2-4-6, "v"	1-2-4-6, "w"	1-2-4-6, "x"	1-2-4-6, "y"	1-2-4-6, "z"	4-6, 1-2-4-6	1-2-4-6, 2-4-6
				⠫⠵	⠨⠫	⠫⠪
				⏢	▽	∠
				trapeZoid	inverted triangle	angle

1-2-4-6 SIGN FOLLOWED BY NEMETH NUMERALS TO SHOW NUMBER OF SIDES

1-2-4-6, 2-5-6	1-2-4-6, 2-6	1-2-4-6, 2-3-5	1-2-4-6, 2-3-5-6	1-2-4-6, 2-3-6	1-2-4-6, 3-5	(etc.)
⠫⠲	⠫⠢	⠫⠖	⠫⠶	⠫⠦	⠫⠔	
						
square 4-sided	pENtagon 5-sided	hexagon 6-sided	heptagon 7-sided	octagon 8-sided	nonagon 9-sided	

Shapes can be modified by following the 1-2-4-6 shape indicator with cues: 4-5-6 is for filled-in symbols; 4-6 is for shaded-in symbols. The 3-4 sign before the 1-2-4-6 sign indicates "not."

COMMONLY USED FOREIGN LANGUAGE LETTERS

These are shortened charts for some of the foreign languages and some of the alphabetic characters. Refer to The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revison, pages 20-25 for a comprehensive chart.

GREEK LETTERS (STANDARD)					
CAPITALIZED			LOWER CASE		
LETTER	NAME	BRAILLE	LETTER	NAME	BRAILLE
Α	ALPHA	⠨⠠⠁	α	alpha	⠨⠁
Β	BETA	⠨⠠⠃	β	beta	⠨⠃
Γ	GAMMA	⠨⠠⠛	γ	gamma	⠨⠛
Δ	DELTA	⠨⠠⠙	δ	delta	⠨⠙
Ε	EPSILON	⠨⠠⠑	ϵ	epsilon	⠨⠑
Θ	THETA	⠨⠠⠹	θ	theta	⠨⠹
Λ	LAMBDA	⠨⠠⠇	λ	lambda	⠨⠇
Π	PI	⠨⠠⠏	π	pi	⠨⠏
Ρ	RHO	⠨⠠⠗	ρ	rho	⠨⠗
Σ	SIGMA	⠨⠠⠎	σ	sigma	⠨⠎
Υ	UPSILON	⠨⠠⠥	υ	upsilon	⠨⠥
Φ	PHI	⠨⠠⠋	ϕ	phi	⠨⠋
Ω	OMEGA	⠨⠠⠺	ω	omega	⠨⠺

RUSSIAN (CYRILLIC) ALPHABET					
CAPITALIZED			LOWER CASE		
LETTER	NAME	BRAILLE	LETTER	NAME	BRAILLE
А	RUSSIAN AH	⠈⠈⠠⠁	а	RUSSIAN ah	⠈⠈⠁
Б	RUSSIAN BEH	⠈⠈⠠⠃	б	RUSSIAN beh	⠈⠈⠃
В	RUSSIAN VEH	⠈⠈⠠⠺	в	RUSSIAN veh	⠈⠈⠺
Г	RUSSIAN GHEH	⠈⠈⠠⠛	г	RUSSIAN gheh	⠈⠈⠛

HEBREW		
ORDINARY		
LETTER	NAME	BRAILLE
א	Aleph	⠠⠠⠁
ב	Veth	⠠⠠⠧
ג	Gimel	⠠⠠⠛
ח	Cheth	⠠⠠⠭
ם	Mem	⠠⠠⠍

SUMMARY

SYMBOLS UNIQUE TO BRAILLE		
NAME	NUMERIC INDICATOR	PUNCTUATION INDICATOR
PRINT SYMBOL	NONE unique to braille	NONE unique to braille
BRAILLE SYMBOL	⠼	⠸
CONFIG-URATION	dots 3-4-5-6	dots 4-5-6

(1) The numeric indicator: place before a numeral at the beginning of a line or after a space. (Lesson 1)
(2) The punctuation indicator: place between a mathematical symbol and the punctuation mark (list: ? . : ; ' "); do not use with (list: , - --). Use with: ____ long dash.
(Refer to Lesson 1.5)

SUMMARY

PUNCTUATION MARKS REQUIRING THE PUNCTUATION INDICATOR				
NAME	PERIOD	QUESTION MARK	COLON	SEMICOLON
PRINT SYMBOL	.	?	:	;
BRAILLE SYMBOL	⠲	⠦	⠒	⠆
CONFIG-URATION	dots 2-5-6	dots 2-3-6	dots 2-5	dots 2-3

(1) Use these with the punctuation indicator. (Lesson 1)
(2) Space after the mark of punctuation.
(3) Do NOT use this colon to represent the print mathematical symbol indicating ratio.
(Refer to Lesson 1)

SUMMARY

PUNCTUATION REQUIRING THE PUNCTUATION INDICATOR
(continued)

NAME	APOSTROPHE	EXCLAMATION POINT	OPENING QUOTES (DOUBLE)	CLOSING QUOTES (DOUBLE)
PRINT SYMBOL	‘	!	“	”
BRAILLE SYMBOL	⠄	⠖	⠦	⠴
CONFIG-URATION	dot 3	dots 2-3-5	dots 2-3-6	dots 3-5-6

(1) Use these marks with the punctuation indicator (dots 4-5-6).
(2) Space after the mark of punctuation.
(3) Do NOT use the exclamation point to represent the print mathematical symbol used to indicate the factorial (use dots 1-2-3-4-6 for the factorial).
(Refer to Lesson 1.5)

SUMMARY

PUNCTUATION NOT REQUIRING THE PUNCTUATION INDICATOR			
NAME	COMMA	HYPHEN	DASH
PRINT SYMBOL	,	-	--
BRAILLE SYMBOL	⠠	⠤	⠤⠤
CONFIG-URATION	dot 6	dots 3-6	dots 3-6, 3-6

(1) The comma: use to separate place values, in lists, with co-ordinates, as a mark of punctuation for mathematical items; do NOT use the punctuation indicator. (Lesson 1)
(2) Use dash without the punctuation indicator (exception: the punctuation indicator is needed when a long dash is used in a mathematical context, e.g., as a sign of omission; NBC §37iv).
(Refer to Lesson 1.2)

SUMMARY

SIGNS OF OPERATION				
NAME	PLUS	MINUS		
PRINT SYMBOL	+	-		
BRAILLE SYMBOL	⠬	⠤		
CONFIG-URATION	dots 3-4-6	dots 3-6		

(1) When it indicates a mathematical operation: do NOT space between the sign and the symbols on which it is operating; do NOT use the numeric indicator between it and the symbols on which it is operating.
(2) When it indicates a signed number: insert the numeric indicator between the minus sign (-) and the symbol to which it applies IF it occurs as the first symbol on a braille line or after a space.
(3) Insert a dot 5 between two adjacent signs (e.g., one is used as a sign of operation and the other is used to indicate a signed number).
(4) Punctuate as a mathematical symbol.
(Refer to Lesson 2.)

SUMMARY

SIGNS USED IN SPATIAL DISPLAYS:
CARRIED NUMBER AND CANCELLATION INDICATORS

NAME	CARRIED NUMBER INDICATORS	OPENING CANCELLATION INDICATOR	CLOSING CANCELLATION INDICATORS
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠶⠶⠶⠶	⠪	⠻
CONFIG-URATION	row of dots 2-3-5-6	dots 2-4-6	dots 1-2-4-5-6

(1) The row of carried number indicators is used with addition problems where regrouping is demonstrated. The row of indicators is the same length as the separation line. The regrouped numbers are placed above this line.
(2) The cancellation indicators are used to enclose the numbers which are cancelled with a slash in the print display (e.g., $\cancel{9}$).
(3) The cancellation indicators generally are unspaced from the numbers to which they apply, from another set of indicators, or from adjacent numbers to the left or right.
(4) Spaces must be left between indicators and the numbers to which they apply IF the regrouped values above consist of more digits than the cancelled value.
(5) When the cancellation indicators are used, alignment in the balance of the problem must be achieved.
(Refer to Lesson 6.)

SUMMARY

SIGNS OF MULTIPLICATION			
NAME	DOT	CROSS	OPENING AND CLOSING PARENTHESIS
PRINT SYMBOL	•	x	( )
BRAILLE SYMBOL	⠡	⠈⠡	⠷ ⠾
CONFIG-URATION	dots 1-6	(two-cell) dots 4, 1-6	dots 1-2-3-5-6 opening dots 2-3-4-5-6 closing

(1) Do not space before or after the sign unless a natural space or line beginning occurs.
(2) Do not use the numeric indicator for numbers enclosed in the parentheses--when multiplication is indicated.
(3) Punctuate as a mathematical symbol.
(Refer to Lesson 4.)

SUMMARY

SIGNS OF DIVISION				
NAME	DIVIDED BY	DIAGONAL FRACTION	HORIZONTAL BAR LINE	CURVED DIVISION
PRINT SYMBOL	÷	/	—	$\overline{)\quad}$
BRAILLE SYMBOL	⠨⠌	⠸⠌	⠌	⠕
CONFIG-URATION	(two-cell) dots 4-6, 3-4	(two-cell) dots 4-5-6, 3-4	dots 3-4	dots 1-3-5

(1) Do not space before or after the signs unless a natural space or line beginning occurs.
(2) The curved division sign should be used without the separation line If the division contains only a divisor and a dividend but no partial differences and no quotient above it.
(3) The horizontal bar line should be used with fractions.
(4) Punctuate these signs of operation as mathematical symbols
(Refer to Lessons 5.1, 5.2, and 5.3)

SUMMARY

SIGNS USED IN SPATIAL DISPLAYS FOR DIVISION				
NAME	RIGHT CURVED DIVISION	LEFT CURVED DIVISION	STRAIGHT DIVISION	SEPARATION LINE
PRINT SYMBOL	⟌	‾(	┌	———
BRAILLE SYMBOL	⠒⠒⠒ ⠕	⠒⠒⠒ ⠪	⠒⠒⠒ ⠸	⠒⠒⠒⠒⠒
CONFIG-URATION	dots 1-3-5 (a row of dots 2-5 is positioned above)	dots 2-4-6 (a row of dots 2-5 is positioned above)	dots 4-5-6 (a row of dots 2-5 is positioned above)	dots 2-5, 2-5, ... (a row of dots 2-5)

(1) In spatial displays, the numeric indicator is NOT used before the divisor.
(2) The separation line above the dividend begins in the same cell as the right curved division sign and extends one cell beyond the overall arrangement of the problem.
(3) If the left curved division sign is used, the separation line extends directly over it.
(4) All separation lines, including those separating the partial differences, are of the same length.
(5) In addition, subtraction, and multiplication problems, the separation line extends one cell to the left and one cell to the right of the longest line in the problem.
(6) Problems arranged spatially are preceded and followed by a blank row unless they are the first or last item on a braille page.
(Refer to Lesson 6.)

SUMMARY

SIGNS USED IN SPATIAL DISPLAYS: CARET AND REMAINDERS

NAME	CARET	REMAINDER (using upper case "R")	REMAINDER (using lower case "r")
PRINT SYMBOL	^	R	r
BRAILLE SYMBOL	⠸⠣	⠠⠗⠐	⠗⠐
CONFIG-URATION	(two cells) dots 4-5-6, 1-2-6	dots 6, 1-2-3-5, 5, and the remainder numeral	dots 1-2-3-5, 5, and the remainder numeral

(1) Except for separation lines, leave corresponding columns in the problem blank. Leave two columns in the partial products section, if necessary, to achieve alignment.
(2) The capital "R" or lower case "r" (to indicate a remainder) is separated from the quotient by a space. The multipurpose indicator (dot 5) is inserted btween the letter and the numeral comprising the remainder. Extend the separation line(s) one cell past the last digit of the remainder
(Refer to Lesson 6.)

SUMMARY

SIGNS OF OPERATION				
NAME	PLUS-OR-MINUS	PLUS MINUS	MINUS PLUS	MINUS MINUS
PRINT SYMBOL	±	+-	-+	--
BRAILLE SYMBOL	⠬⠤	⠬⠐⠤	⠤⠐⠬	⠤⠐⠤
CONFIGURATION	(two-cell) dots 3-4-6, 3-6	(three-cell) dots 3-4-6, 5, 3-6	(three-cell) dots 3-6, 5, 3-4-6	(three-cell) dots 3-6, 5, 3-6

(1) Do NOT space between a sign of operation and the mathematical symbols or numbers associated with it.
(2) Do NOT use the numeric indicator between the sign of operation and the number or symbol following it (except when a number follows a minus minus sign, as in: --3).
(3) Punctuate as mathematical symbols, using the punctuation indicator before all marks of punctuation other than the comma, hyphen, and dash.
(Refer to Lesson 8.)

SUMMARY

SIGNS OF OPERATION

NAME	PLUS PLUS	RADICAL	TERMINATION INDICATOR
PRINT SYMBOL	++	$\sqrt{}$	(unique to braille)
BRAILLE SYMBOL	⠬⠐⠬	⠜	⠻
CONFIG-URATION	(three-cell) dots 3-4-6, 5, 3-4-6	dots 3-4-5	dots 1-2-4-5-6

(1) Do NOT space between a sign of operation and the mathematical symbols or numbers associated with it.
(2) Do NOT use the numeric indicator between the sign of operation and the number or symbol following it.
(3) Punctuate as mathematical symbols, using the punctuation indicator before all marks of punctuation other than the comma, hyphen, and dash.
(4) The radical (root) and termination indicator should be in pairs when a vinculum and radicand are shown in print.
(Refer to Lesson 8.)

SUMMARY

SIGNS OF OPERATION				
NAME	FACTORIAL	INTERSECTION	UNION	HOLLOW DOT
PRINT SYMBOL	!	$\cap$	$\cup$	$\circ$
BRAILLE SYMBOL	⠯	⠨⠩	⠨⠬	⠨⠯
CONFIG-URATION	dots 1-2-3-4-6	(two cell) dots 4-6, 1-4-6	(two cell) dots 4-6, 3-4-6	(two cell) dots 1-2-3-4-6

(1) Do NOT space between a sign of operation and the mathematical symbols or numbers associated with it.
(2) Do NOT use the <u>numeric indicator</u> between the sign of operation and the number or symbol following it.
(3) Punctuate as mathematical symbols, using the punctuation indicator before all marks of punctuation other than the comma, hyphen, and dash.
(4) The hollow dot is also used as a symbol for degree when it is superscripted (refer to Lesson 3).
(Refer to Lesson 8.)

SUMMARY

SIGNS OF OPERATION				
NAME	ASTERISK			
PRINT SYMBOL	*			
BRAILLE SYMBOL	⠈⠼			
CONFIG-URATION	(two-cell) dots 4, 3-4-5-6			

When used as a sign of operation:
(1) do not space between the asterisk and the mathematical symbols or numbers associated with it,
(2) place the numeric indicator between the asterisk and a numeral. or decimal point and numeral, which follows, (do not use the numeric indicator with any other symbol following the asterisk),
(3) punctuate as a mathematical symbol, and
(4) if the asterisk symbol is at a different level from the material associated with it, then the appropriate level indicator must be used to indicate its position.

The asterisk is also used as a reference symbol. (Refer to Lesson 8.4 for both usages) (Refer to Lesson 8.)

SUMMARY

SIGNS OF COMPARISON

NAME	EQUAL		
PRINT SYMBOL	=		
BRAILLE SYMBOL	⠨⠅		
CONFIG-URATION	(two-cell) dots 4-6, 1-3		

(1) In general, space before and after the symbol, except when the symbols occur in pairs or with a negation slash (Lesson 2.2 and Lesson 11).
(2) Use the numeric indicator for numerals occurring immediately AFTER the sign of comparison.
(3) Punctuate as a mathematical symbol: DO NOT space between the sign and the punctuation indicator (dots 4-5-6).
(Refer to Lesson 2.2.)

SUMMARY

SIGNS OF OMISSION			
NAME	ELLIPSIS	OMISSION (General)	OMISSION LONG DASH
PRINT SYMBOL	...	? (OR A BLANK SPACE)	———
BRAILLE SYMBOL	⠄⠄⠄	⠿	⠤⠤⠤⠤
CONFIG-URATION	(three-cell) dots 3, 3, 3	dots 1-2-3-4-5-6	(four cell) 3-6, 3-6, 3-6, 3-6

(1) Space between an ellipsis or long dash and the adjacent symbols with which it is associated (even with signs of operation and others that normally do not require a space).

Exceptions: do not space between it and:

- adjacent punctuation (other than the hyphen). These symbols are to be punctuated as mathematical symbols when used mathematically and the long dash and ellipsis punctuated according to literary format when used in a literary context.
- a related monetary symbol ($, ¢, £), decimal, percent, or prime sign.
- braille indicators (numeric indicator, Enlish letter indicator, etc.)
- with symbols of grouping (such as parentheses, brackets, etc.; refer to Lesson 10)

(2) These symbols occupy the same position in braille as they do in print.

(3) Use the general omission symbol (the full braille cell) as it would be used for the symbol it replaces.

(4) If a question mark appears over a blank line or with dashes on either side, use only the general omission symbol.

(Lesson 3.3 and §57 NBC)

SUMMARY

SIGNS OF COMPARISON: CARD A				
NAME	EQUAL	NOT EQUAL	SIMILAR TO	CONGRUENT
PRINT SYMBOL	$=$	$\neq$	$\sim$	$\cong$
BRAILLE SYMBOL	⠨⠅	⠌⠨⠅	⠈⠱	⠈⠱⠨⠅
CONFIG-URATION	dots 4-6, 1-3	dots 3-4, 4-6, 1-3	dots 4, 1-5-6	dots 4, 1-5-6, 4-6, 1-3

(1) Space before and after the sign of comparison. A space must be left (on both sides) between a sign of comparison and any other mathematical symbols or expressions which precede or follow it--refer to Rule 2 for exceptions.

a) The space requirement also applies when words are used with the comparison sign, as in: three+seven = ten.

b) Contractions and short-form words may not be used before or after The space that precedes or follows the sign of comparison (§55 NBC).

(2) Do not space between a sign of comparison and a mark of punctuation, a braille indicator (e.g., italic font indicator, etc.), or a sign of grouping that is associated with it. The use or non-use of the punctuation indicator (dots 4-5-6) follows the rules which cover the application of that symbol.

(3) Punctuate as a mathematical symbol.

(4) When a long problem must be divided between braille lines (avoid dividing a problem, expression, or compared expressions, if possible):

a) the split should occur before the sign of comparison,

b) in embedded material, the runover portion is placed on the next braille line at the margin of the material preceding it,

c) in displayed material, the runover portion is placed on the next braille line and is indented two cells from the margin of the material preceding it.

(5) For material following the space after a sign of comparison, use the numeric indicator according to the rules which apply to it.

(Refer to Lesson 11 and Lesson 12.)

SUMMARY

SIGNS OF COMPARISON: CARD B				
NAME	EQUALS APPROX-IMATELY	EQUIVALENT (IDENTITY)	RATIO "IS TO"	PROPORTION
PRINT SYMBOL	$\approx$	$\equiv$	:	::
BRAILLE SYMBOL	⠈⠱⠈⠱	⠸⠇	⠐⠂	⠰⠆
CONFIG-URATION	dots 4, 1-5-6, 4, 1-5-6	dots 4-5-6, 1-2-3	dots 5, 2	dots 5-6, 2-3

(1) Space before and after the sign of comparison. A space must be left (on both sides) between a sign of comparison and any other <u>mathematical</u> symbols or expressions which precede or follow it--refer to Rule 2 for exceptions.

a) The space requirement also applies when words are used with the comparison sign, as in: three+seven = ten.

b) Contractions and short-form words may not be used before or after The space that precedes or follows the sign of comparison (§55 NBC).

(2) <u>Do not</u> space between a sign of comparison and a mark of punctuation, a braille indicator (e.g., italic font indicator, etc.), or a sign of grouping that is associated with it. The use or non-use of the punctuation indicator (dots 4-5-6) follows the rules which cover the application of that symbol.

(3) Punctuate as a mathematical symbol.

(4) When a long problem <u>must</u> be divided between braille lines (avoid dividing a problem, expression, or compared expressions, if possible):

a) the split should occur before the sign of comparison,

b) in embedded material, the runover portion is placed on the next braille line at the margin of the material preceding it,

c) in displayed material, the runover portion is placed on the next braille line and is indented two cells from the margin of the material preceding it.

(5) For material following the space after a sign of comparison, use the numeric indicator according to the rules which apply to it.

(Refer to Lesson 11 and Lesson 12.)

SUMMARY

SIGNS OF COMPARISON: CARD C				
NAME	GREATER THAN	LESS THAN	GREATER THAN OR EQUAL TO	LESS THAN OR EQUAL TO
PRINT SYMBOL	$>$	$<$	$\geq$	$\leq$
BRAILLE SYMBOL	⠨⠂	⠐⠅	⠨⠂⠱	⠐⠅⠱
CONFIG-URATION	dots 4-6, 2	dots 5, 1-3	dots 4-6, 2, 1-5-6	dots 5, 1-3, 1-5-6

(1) Space before and after the sign of comparison. A space must be left (on both sides) between a sign of comparison and any other mathematical symbols or expressions which precede or follow it--refer to Rule 2 for exceptions.

a) The space requirement also applies when words are used with the comparison sign, as in: three+seven = ten.

b) Contractions and short-form words may not be used before or after The space that precedes or follows the sign of comparison (§55 NBC).

(2) Do not space between a sign of comparison and a mark of punctuation, a braille indicator (e.g., italic font indicator, etc.), or a sign of grouping that is associated with it. The use or non-use of the punctuation indicator (dots 4-5-6) follows the rules which cover the application of that symbol.

(3) Punctuate as a mathematical symbol.

(4) When a long problem must be divided between braille lines (avoid dividing a problem, expression, or compared expressions, if possible):

a) the split should occur before the sign of comparison,

b) in embedded material, the runover portion is placed on the next braille line at the margin of the material preceding it,

c) in displayed material, the runover portion is placed on the next braille line and is indented two cells from the margin of the material preceding it.

(5) For material following the space after a sign of comparison, use the numeric indicator according to the rules which apply to it.

(Refer to Lesson 11 and Lesson 12.)

SUMMARY

SIGNS OF COMPARISON: CARD D (set theory notation)				
NAME	ELEMENT OF	PROPER SUBSET	SUBSET (INCLUSION)	SUPERSET (REVERSE INCLUSION)
PRINT SYMBOL	$\in$	$\subset$	$\subseteq$	$\supset$
BRAILLE SYMBOL	⠈⠑	⠸⠐⠅	⠸⠐⠅⠱	⠸⠨⠂
CONFIG-URATION	dots 4, 1-5	dots 4-5-6, 5, 1-3	dots 4-5-6, 5, 1-3, 1-5-6	dots 4-5-6, 4-6, 2

(1) Space before and after the sign of comparison. A space must be left (on both sides) between a sign of comparison and any other <u>mathematical</u> symbols or expressions which precede or follow it--refer to Rule 2 for exceptions.

a) The space requirement also applies when words are used with the comparison sign, as in: three+seven = ten.

b) Contractions and short-form words may not be used before or after the space that precedes or follows the sign of comparison (§55 NBC).

(2) <u>Do not</u> space between a sign of comparison and a mark of punctuation, a braille indicator (e.g., italic font indicator, etc.), or a sign of grouping that is associated with it. The use or non-use of the punctuation indicator (dots 4-5-6) follows the rules which cover the application of that symbol.

(3) Punctuate as a mathematical symbol.

(4) When a long problem <u>must</u> be divided between braille lines (avoid dividing a problem, expression, or compared expressions, if possible):

a) the split should occur before the sign of comparison,

b) in embedded material, the runover portion is placed on the next braille line at the margin of the material preceding it,

c) in displayed material, the runover portion is placed on the next braille line and is indented two cells from the margin of the material preceding it.

(5) For material following the space after a sign of comparison, use the numeric indicator according to the rules which apply to it.

(Refer to Lesson 11 and Lesson 12.)

SUMMARY

SIGNS OF GROUPING

NAME	OPENING PARENTHESIS	CLOSING PARENTHESIS	OPENING BRACKET	CLOSING BRACKET
PRINT SYMBOL	(	)	[	]
BRAILLE SYMBOL	⠷	⠾	⠈⠷	⠈⠾
CONFIG-URATION	dots 1-2-3-5-6	dots 2-3-4-5-6	(two-cell) dots 4, 1-2-3-5-6	(two-cell) dots 4, 2-3-4-5-6

(1) Signs of grouping should appear in the same position as they do in print.
(2) No one-cell whole-word alphabet (e.g., but, can, do, etc.), whole or part word (e.g., of, the, for, and, etc.), or whole-word lower-sign (e.g., enough, was, in, by, to, etc.) contractions may be in contact with these symbols.
(3) The use or non-use of the numeric indicator or English letter indicator depends upon the rules for those symbols.
(4) Punctuate as mathematical symbols.
(5) Other rules for enlarged, double, and boldface symbols and for the use of braille contractions may apply.
(Refer to Lesson 10 and to the sections containing the specific sign of grouping.)

SUMMARY

SIGNS OF GROUPING				
NAME	OPENING BRACE	CLOSING BRACE	OPENING VERTICAL BAR LINE	CLOSING VERTICAL BAR LINE
PRINT SYMBOL	{	}				
BRAILLE SYMBOL	⠨⠷	⠨⠾	⠳	⠳
CONFIG-URATION	(two-cell) dots 4-6, 1-2-3-5-6	(two-cell) dots 4-6, 2-3-4-5-6	dots 1-2-5-6	dots 1-2-5-6

(1) Signs of grouping should appear in the same position as they do in print.
(2) No one-cell whole-word alphabet (e.g., but, can, do, etc.), whole or part word (e.g., of, the, for, and, etc.), or whole-word lower-sign (e.g., enough, was, in, by, to, etc.) contractions may be in contact with these symbols.
(3) The use or non-use of the numeric indicator or English letter indicator depends upon the rules for those symbols.
(4) Punctuate as mathematical symbols.
(5) Other rules for enlarged, double, and boldface symbols and for the use of braille contractions may apply.
(Refer to Lesson 10 and to the sections containing the specific sign of grouping.)

SUMMARY

TRANSCRIBER'S GROUPING SYMBOLS

NAME	TRANSCRIBER'S GROUPING SYMBOLS
PRINT SYMBOL	(none) unique to braille
BRAILLE SYMBOL	⠠⠄
CONFIG-URATION	dots 6, 3 (two cells)

(1) Use to enclose a key (to open and close as a sign of grouping) used for figure labels, column headings, table entries, etc.
(2) A key must be enclosed in transcriber's grouping symbols. The key is preceded by the transcriber's grouping symbol. It is indented six cells in from the margin with runovers indented four cells. After a blank line, key *entries* begin at the margin with runovers indented two cells. The final entry in the key must be followed by the transcriber's grouping symbol.
(3) The transcriber's symbol is also used to enclose explanations of seven or fewer words at a site next to the text being explained.
(4) When transcriber's notes are longer than seven words, the first character of the transcriber's grouping symbol is indented six cells from the margin of the material directly preceding the note, followed by the material in the note. Runovers are indented four cells.
(Refer to Lesson §16.2)

SUMMARY

FRACTION INDICATORS				
NAME	SIMPLE OPENING	SIMPLE CLOSING	MIXED-NUMBER OPENING	MIXED-NUMBER CLOSING
PRINT SYMBOL	unique to braille	unique to braille	unique to braille	unique to braille
BRAILLE SYMBOL	⠹	⠼	⠸⠹	⠸⠼
CONFIG-URATION	dots 1-4-5-6	dots 3-4-5-6	(two-cell) dots 4-5-6, 1-4-5-6	(two-cell) dots 4-5-6, 3-4-5-6

(1) Use these to enclose fractions; the mixed-number indicators only enclose the fractional part of the number.
(2) Use the horizontal division bar line to separate the numerator and denominator.
(3) Use the mixed-number indicators for expressions which begin with a numeral and are followed by a simple fraction; an expression is not a mixed number if it contains any letter (even if it is in the same form).
(4) Punctuate as mathematical symbols.
(Refer to Lesson 5.)

SUMMARY

COMPLEX FRACTION INDICATORS: TABLE A

NAME	OPENING COMPLEX FRACTION INDICATOR	CLOSING COMPLEX FRACTION INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠠⠹	⠠⠼
CONFIG- URATION	(two cells) dots 6, 1-4-5-6	(two cells) dots 6, 3-4-5-6

(1) these symbols are used in pairs and begin and end the portion of an expression that is a complex fraction.
(2) the complex horizontal fraction line or diagonal complex fraction line (slash) is used to separate the numerator and denominator of the complex fraction. (Refer to Table B.)
(3) the symbols may be used in a linear (horizontal) display or in a spatial (vertical) display.
(Refer to Lesson 17.1)

SUMMARY

COMPLEX FRACTION INDICATORS: TABLE B		
NAME	HORIZONTAL COMPLEX FRACTION LINE	DIAGONAL COMPLEX FRACTION LINE (SLASH)
PRINT SYMBOL	(elongated horizontal line) ———	(elongated fraction slash) /
BRAILLE SYMBOL	⠠⠌	⠠⠸⠌
CONFIG-URATION	(two cells) dots 6, 3-4	(two cells) dots 6, 4-5-6, 3-4

(1) these symbols are used to separate the numerator and denominator of the complex fraction.
(2) the symbols are used in relation to the opening and closing complex fraction indicators in horizontal displays.
(3) in spatial displays, the complex horizontal fraction line is converted to become a row of dots 2-5 (of varying length) preceded and followed by the opening and closing complex fraction indicators.
(Refer to Lesson 17.1)

SYMBOLS FOR HYPERCOMPLEX FRACTIONS: TABLE A		
NAME	OPENING HYPERCOMPLEX FRACTION INDICATOR	CLOSING HYPERCOMPLEX FRACTION INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠠⠠⠹	⠠⠠⠼
CONFIG-URATION	(three cells) dots 6, 6, 1-4-5-6	(three cells) dots 6, 6, 3-4-5-6

(1) These two symbols are used in pairs.
(2) In spatial and mixed linear-and-spatial displays, they begin and end the fraction line (row of dots 2-5). Refer to Table B.
(3) In linear (horizontal) display, they mark the beginning and the end of a hypercomplex fraction.
(Refer to Lesson 17.2)

SUMMARY

SYMBOLS FOR HYPERCOMPLEX FRACTIONS: TABLE B

NAME	HYPERCOMPLEX HORIZONTAL FRACTION LINE	DIAGONAL HYPERCOMPLEX FRACTION LINE
PRINT SYMBOL	(elongated fraction line) ————————	(elongated fraction slash) /
BRAILLE SYMBOL	(Spatially: varies in length) ⠠⠠⠹⠒⠒⠒⠒⠠⠠⠼ or (linearly) ⠠⠠⠌	⠠⠠⠸⠌
CONFIG-URATION	(varies in length) dots 6, 6, 1-4-5-6, 2-5, 2-5, ..., 6, 6, 3-4-5-6 or dots 6, 6, 3-4	(four cells) dots 6, 6, 4-5-6, 3-4

(1) The complex horizontal fraction line of varying length is used in a spatial display (or mixed linear-and-spatial display) to represent the horizontal hypercomplex fraction line.
(2) The dots 2-5 portion of the hypercomplex fraction line is the same length as the longest expression (in either the numerator or the denominator). The numerator and denominator are centered in relation to this line.
(3) the three-celled hypercomplex fraction line is used only in linear (horizontal) displays.
(4) The diagonal hypercomplex fraction line is used in a linear (horizontal) display when the print shows a slash fraction line separating the complex fraction portion from the remaining section.
(Refer to Lesson 17.)

SUMMARY

GRADIENT SYMBOLS

NAME	DEGREE	MINUTES, FEET, PRIME	SECONDS, INCHES
PRINT SYMBOL	°	′	″
BRAILLE SYMBOL	⠘⠨⠡	⠄	⠄⠄
CONFIG-URATION	(three cell*) dots 4-5, 4-6, 1-6	dot 3	(two-cell) dots 3, 3

(1) Place in the same position as the corresponding print symbol.
(2) Do NOT space between these symbols and adjacent symbols.
(3) Punctuate as mathematical symbols.
(4) For the degree sign only:
a) the superscript indicator MUST be used UNLESS the symbol stands apart from other symbols,
b) if an abbreviation for the temperature scales of Fahrenheit and Centigrade are used following the degree symbol, space after the degree symbol and use the English letter indicator (dots 4-5) and the capitalization indicator (dot 6) before the letter "F" or "C".
*c) If a sign of operation or other symbol immediately follows the degree, insert a baseline indicator (dot 5) between the degree symbol and the adjacent symbol.
(Lesson §3.7 & §3.8)

SUMMARY

DECIMAL AND MONETARY SYMBOLS				
NAME	DECIMAL POINT	DOLLAR SIGN	CENT SIGN	POUND STERLING
PRINT SYMBOL	.	$	¢	£
BRAILLE SYMBOL	⠨	⠈⠎	⠈⠉	⠈⠇
CONFIG-URATION	dots 4-6	(two-cell) dots 4, 2-3-4	(two-cell) dots 4, 1-4	(two-cell) dots 4, 1-2-3

(1) Place in corresponding position as it would appear in print in both horizontal and vertical-spatial presentations.
(2) Do NOT leave a space between these symbols and the numbers or mathematical symbols they pertain to.
(3) These are mathematical symbols and should be punctuated accordingly. (Refer to Lesson 3.)

SUMMARY

PERCENT, INFINITY, AND EMPTY SET			
NAME	PERCENT	INFINITY	EMPTY SET
PRINT SYMBOL	%	∞	$\varnothing$
BRAILLE SYMBOL	⠈⠴	⠠⠿	⠸⠴
CONFIG-URATION	(two-cell) dots 4, 3-5-6	(two-cell) dots 6, 1-2-3-4-5-6	(two-cell) dots 4-5-6, 3-5-6

(1) Do NOT space between the symbol and the adjacent symbols with which it is associated.
(2) These symbols occupy the same position in braille as they do in print.
(3) They are to be punctuated as mathematical symbols.
(Refer to Lesson 3.)

SUMMARY

SPECIAL SYMBOLS		
NAME	TALLY MARK	AT SIGN
PRINT SYMBOL	|	@
BRAILLE SYMBOL	⠸	⠈⠁
CONFIG-URATION	dots 4-5-6	(two-cell) dot 4, dot 1

(1) Use the same number of tally marks as are displayed in print; if a fifth cross tally mark is displayed, use an extra (fifth) braille tally mark. An intervening space is used to separate tally marks.
(2) When a tally mark is used with a mark of punctuation which requires the punctuation indicator, insert the multipurpose indicator before the punctuation indicator.
(3) Punctuate the ***at*** sign (@) as a mathematical symbol.
(4) In ALL cases, other than when both marks are used with puntuation or signs of grouping or related indicators, both marks are to be preceded and followed by a space.
(Refer to Lesson 7.7).

SUMMARY

LEVEL INDICATORS

NAME	SUPERSCRIPT INDICATOR	SUBSCRIPT INDICATOR	BASELINE INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠘	⠰	⠐
CONFIG-URATION	dots 4-5	dots 5-6	dot 5

(1) For first order superscripts: precede the portion that is raised by using the superscript level indicator.
(2) For first order subscripts: use with non-numeric subscripts, material subscripted to a baseline numeral, left numeric subscripts; do not use with right numeric subscripts to letters or abbreviated function names.
(3) For levels other than first order: use as many level indicators as the level on which the item is placed; return to a reduced level with the number of indicators appropriate to that level. Return to the baseline with the baseline indicator.
(4) The baseline indicator is used to show a return to the base line of print when material not associated with the superscripted or subscripted items follows.
(5) The influence of level indicators is terminated by a space, another level indicator, the baseline indicator, a punctuation indicator, or a transition to a new braille line.
(Refer to Lesson 12.)

SUMMARY

COMBINED LEVEL INDICATORS		
NAME	COMBINED SUPERSCRIPT AND SUBSCRIPT	COMBINED SUBSCRIPT AND SUPERSCRIPT
PRINT SYMBOL	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠘⠰	⠰⠘
CONFIG-URATION	(two cells) dots 4-5, 5-6	(two cells) dots 5-6, 4-5

(1) the first level change is handled with a basic level indicator appropriate for that level.
(2) the first cell of the combined level indicator indicates the position relative to the baseline material; the second cell indicates the position relative to the superscripted or subscripted material being supplemented.
(3) other multiple indicators can be created in a similar fashion.
(Refer to Lesson 12.4.)

SUMMARY

RADICAL MODIFIERS

NAME	INDEX-OF-RADICAL INDICATOR	ORDER-OF-RADICAL INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠣	⠨
CONFIG-URATION	dots 1-2-6	dots 4-6

(1) The index-of-radical indicator is used preceding a value which in print is tucked into the crook of a radical indicator. It is followed by the value which is the index, the radical indicator, radicand, etc. (Refer to Lesson 13.1.)
(2) The order-of-radical indicator is used when radicals are nested within other radicals. The indicator precedes the radical indicator (or the index-of-radical-indicator if one is present) of the imbedded radical AND precedes the termination indicator for the radical. Each imbedded radical requires the same number of order-of-radical indicators as its level of nesting.
(Refer to Lesson 13.2.)

SUMMARY

SIGNS USED WITH SHAPES

NAME	SHAPE INDICATOR	MULTIPURPOSE INDICATOR	TERMINATION INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠫	⠐	⠻
CONFIG-URATION	dots 1-2-4-6	dot 5	dots 1-2-4-5-6

(1) shape indicator precedes letter, character, or numeral which identifies the shape.

(2) multipurpose indicator, when applied to shapes:

- precedes material to introduce a modification and to indicate that what follows the material (the shape) is being modified by the directly over or directly under indicators
- follows a regular numbered polygon shape used as a sign of operation when a numeral follows the shape.

(3) termination indicator ends shape modifications.

(Refer to Lesson 14.)

SUMMARY

MODIFICATION INDICATORS USED WITH SHAPES		
NAME	STRUCTURAL SHAPE-MODIFICATION INDICATOR	INTERIOR SHAPE-MODIFICATION INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠨	⠸⠫
CONFIG-URATION	dots 4-6	(two cells) dots 4-5-6, 1-2-4-6

(1) For structural modifications of a basic shape: braille a shape indicator, the basic shape, the shape-modification indicator, character(s) making the alteration, and end with the termination indicator (dots 1-2-4-5-6).
(2) For interior modifications to a basic shape: braille the basic shape (with its shape indicator), the interior shape-modification indicator, the symbol which is interior to the shape, and end with the termination indicator (dots 1-2-4-5-6). (Refer to Lesson 14.4.)

SUMMARY

POSITIONING SHAPES/SHADING SHAPES

NAME	DIRECTLY-OVER INDICATOR	DIRECTLY-UNDER INDICATOR	FILLED-IN SHAPE INDICATOR	SHADED SHAPE INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠣	⠩	⠸	⠨
CONFIG-URATION	dots 1-2-6	dots 1-4-6	dots 4-5-6	dots 4-6

(1) When a shape is positioned above or below material, precede the affected material with the multipurpose indicator (dot 5), braille the affected material. This is followed by the position indicator and the shape (e.g., arrow, line). End with the termination indicator (dots 1-2-4-5-6).
(2) When shaded or filled-in shapes are brailled: braille shape indicator, shaded (or filled-in) indicator, character(s) representing the shape.
(Refer to Lesson 14.4.)

SUMMARY

TYPE-FORM INDICATORS: TABLE A

NAME	BOLDFACE TYPE INDICATOR	ITALIC TYPE INDICATOR	SCRIPT INDICATOR	SANS-SERIF INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠸	⠨	⠈	⠠⠨
CONFIG-URATION	dots 4-5-6	dots 4-6	dot 4	dots 6, 4-6

(1) These indicators are used to show a modification in print fonts.
(2) These are not always used when a change in font takes place (refer to §15.1.1).
(3) When used with numerals, they are placed before the numeric indicator or decimal point. Their influence extends to a space or to a change in font type form.
(4) When used with letters, they are followed by an alphabetic indicator. Their influence extends ONLY to the one letter to the right.
(Refer to Lessons 15.1 and 15.2 and to Summary Table B)

SUMMARY

TYPE-FORM INDICATORS: TABLE B				
NAME	OPENING BOLDFACE TYPE INDICATOR	CLOSING BOLDFACE TYPE INDICATOR	OPENING ITALIC TYPE INDICATOR	CLOSING ITALIC TYPE INDICATOR
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠠⠄⠸	⠸⠠⠄	⠠⠄⠨	⠨⠠⠄
CONFIG-URATION	dots 6, 3, 4-5-6	dots 4-5-6, 6, 3,	dots 6, 3, 4-6	dots 4-6 6, 3,

(1) The italic typeform indicators are only used in the text of labeled statements (NOT the label) and in all material in the general text that begins and/or ends with a mathematical symbol. All other material uses the English braille italics form.
(2) These indicators are used to show a modification of print when nonregular forms of type font are used for mathematical statements, words, and phrases.
(3) Each indicator is preceded and followed by a space.
(4) The appropriate opening indicator is placed before the word or phrase which is in a nonregular form of type; the appropriate closing indicator ends the influence of the modification and follows the last character which is in a nonregular form of type.
(5) These indicators are NOT always used when a change in form of type is displayed in print (e.g., in labeled statements, the label is brailled entirely in capital letters, even if the print displays the label in an italics or boldface font). (Refer to Lesson 15.3 and Summary table A)

SUMMARY

ROMAN NUMERAL INDICATORS

NAME	ENGLISH LETTER	CAPITALIZATION	DOUBLE CAPITALIZATION
PRINT SYMBOL	(none) unique to braille	(none) unique to braille	(none) unique to braille
BRAILLE SYMBOL	⠰	⠠	⠠⠠
CONFIG-URATION	dots 5-6	dot 6	(two-cell) dot 6, dot 6

(1) Use the English letter indicator with lower-case Roman numerals; use the English letter indicator, along with the capitalization indicator, with single capital Roman numerals; both are placed unspaced before the numerals.
(2) The capitalization indicator (single), when used, follows the English letter indicator.
(3) The double capitalization indicator is used before Roman numerals consisting of more than one capitalized letter.
(Refer to Lesson 7.1)

SUMMARY

FOREIGN LANGUAGE LETTER INDICATORS				
NAME	ENGLISH	GREEK (upper case)	GREEK (lower case)	GERMAN
PRINT SYMBOL	unique to braille	unique to braille	unique to braille	unique to braille
BRAILLE SYMBOL	⠰	⠨⠠	⠨	⠸
CONFIG-URATION	dots 5-6	dots 4-6,6	dots 4-6	dots 4-5-6

(1) Precede the letter with the appropriate braille foreign language letter indicator.
(2) The foreign language indicator's influence extends only to the (single) letter which follows it.
(3) The foreign language indicators are used as mathematical symbols and are subject to the rules for such symbols.
(Refer to Lesson 9.3.)

SUMMARY

FOREIGN LANGUAGE LETTER INDICATORS		
NAME	HEBREW	RUSSIAN CYRILLIC
PRINT SYMBOL	(unique to braille)	(unique to braille)
BRAILLE SYMBOL	⠠⠠	⠈⠈
CONFIG-URATION	(two-cells) dot 6, dot 6	(two-cells) dot 4, dot 4

(1) Precede the letter with the appropriate braille foreign language letter indicator.
(2) The foreign language indicator's influence extends only to the (single) letter which follows it.
(3) The foreign language indicators are used as mathematical symbols and are subject to the rules for such symbols.
(Refer to Lesson 9.3.)

SUMMARY

SIGMA NOTATION, LIMITS AND INTEGRALS			
NAME	SIGMA	LIM (also LIMIT)	INTEGRAL
PRINT SYMBOL	$\sum$	lim	$\int$
BRAILLE SYMBOL	⠨⠠⠎	⠇⠊⠍	⠮
CONFIG-URATION	(three cells) dots 4-6, 6, s	three cells (or five) dots 1-2-3, 2-4 1-3-4	dots 2-3-4-6

These methods are used to present material positioned at different levels from the symbol or abbreviation:

(1) Each of these may be modified by material positioned either directly over or under the symbol. These modifications will require the use of the five-step rule for modifications and the directly-over and/or directly-under indicators.

(2) A horizontal line positioned directly over or under the items in this group is not treated as a modification requiring the five-step rule (Lesson 14). The sigma is a letter, however, and it uses dots 1-5-6 to indicate the line (as with other single letters and digits). Limit and integral symbols require that the directly over/under-indicator be placed before the symbol.

(3) The sigma, integral, or **lim** may have material in superscripted and/or subscripted positions. These modifications require the use of appropriate level indicators and baseline indicators. (When a sign of comparison is at the different level from the sigma or integral, it too is preceded by the appropriate level indicator.)

(4) When expressions appear at different levels other than the baseline, the "lower" expression is brailled first.

(5) The expression on the baseline following the sigma or integral is brailled unspaced from the symbol (or the final symbol of its modification).

(6) "lim" is a function name abbreviation and a space follows the name (or the end of a modification to the abbreviation).

(Refer to Lesson 18.)

Common Nemeth Symbols

Operation

- +
- -
- ×
- •
- ÷
- ±
- ∓
- +-
- -+
- --

Comparison

- =
- ≠
- ≅
- ≡
- >
- ≥
- <
- ≤
- ::
- :
- ~
- ≈
- ⊥
- ∥

Grouping

- (
-)
- [
-]
- {
- }
- |
- ‖
- Enlarge, add

Miscellaneous

- , [math comma]
- . [decimal]
- /
- ¢
- $
- %
- √ [radical]
- ° [hollow dot]
- ∫ [integral]
- ! [factorial]
- @
- ∞
- General omission

Indicators

- Punctuation
- English letter
- Greek letter
- Script
- Boldface
- Italic
- Open Bold
- Close Bold
- Open Italic
- Close Italic
- Transcriber's note
- Superscript
- Subscript
- Baseline
- Multipurpose
- Directly over
- Directly under
- Termination
- Simple Fraction:
  - Opening
  - Closing
  - Horizontal line
  - Diagonal line
- Mixed Number:
  - Opening
  - Closing

Shape

- Shape
- ∠
- →
- ←
- ↔
- ○
- □
- ■
- △
- ☆
- ★

Set Theory

- ∪
- ∩
- ⊂
- ⊃
- ∈
- R
- ∋

Reference

- *
- †
- ‡
- ¶
- §
- ☆
- General

Refer to *The Nemeth Code for Mathematics and Science Notation 1972 Revision* for complete information.

Braille to ASCII Conversion

⠁	A	⠝	N	⠿	=	⠯	&	⠼	#
⠃	B	⠕	O	⠷	(	⠂	1	⠜	>
⠉	C	⠏	P	⠮	!	⠆	2	⠄	'
⠙	D	⠟	Q	⠾	)	⠒	3	⠤	-
⠑	E	⠗	R	⠡	*	⠲	4	⠈	@
⠋	F	⠎	S	⠣	<	⠢	5	⠘	^
⠛	G	⠞	T	⠩	%	⠖	6	⠸	_
⠓	H	⠥	U	⠹	?	⠶	7	⠐	"
⠊	I	⠧	V	⠱	:	⠦	8	⠨	.
⠚	J	⠺	W	⠫	$	⠔	9	⠰	;
⠅	K	⠭	X	⠻	]	⠴	0	⠠	,
⠇	L	⠽	Y	⠳	\	⠌	/		
⠍	M	⠵	Z	⠪	[	⠬	+		

Contractions NOT USED:
Before or after
1. comparison
2. operation
3. slash

With Grouping:
- One-cell Alphabet whole word
- Whole-word lower sign
- Whole- or part-word *and, for, of, the, with*

Format
1. Directions 5-3
2. Exercise 1-3
3. Exercise w/subdivisions 1-5, 3-5
4. Display to text 3-5
5. Display to exercise 5-7
6. Display to exercise w/subdivisions 7-9

Refer to *The Nemeth Code for Mathematics and Science Notation 1972 Revision* for complete information.

Five-step Modification
1. Multipurpose indicator
2. Expression
3. Directly over/under indicator
4. Modifying symbol
5. Termination indicator

Prepared 10/96 by:
Buntrock Associates, Inc.

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- Limits
- Integrals

APPENDIX D

RESOURCES

RESOURCES

Internet

http://www.chem.purdue.edu/facilities/sightlab.index.html

VISIONS Lab, Purdue University, Dave Schleppenbach. Use of adaptive technology to produce science and mathematics course information in tactile and/or auditory form; software program that translates mathematics and science equations into a format that can be used by students with visual impairments; this program is available on the Internet.

fttp://fttp.classroom.net/Classroom-Connect/Lessons/ERIC-Plans/Mathematics
fttp://fttp.classroom.net/Classroom-Connect/Lessons/NEW%21-ERIC-Plans/New-Lessons/Mathematics

These sites have some interesting ideas for mathematics lessons.

http://www.coreknowledge.org

This site is the Core Knowledge Home Page with a planned progression of specific knowledge, including mathematics, which some educators believe is a minimum core necessary for people to communicate and progress.

http://people.delphi.com/LUNNEY/ECUPGMS.HTM

Talking Balance 1 is a free software program which enables blind and visually impaired people to operate and read Ohaus electronic balances. It may be downloaded from the World Wide Web at the url listed above.

http://www.japanese-online.com/mathematics/index.htm

This web site provides easy access to mathematics problems translated from Japan's Junior High School mathematics placement test given to 12 year olds. The 225 problems are logic-based and consist of about 20 different types of story problems. This site was developed as a step toward exposure of American students, who recently ranked 14th in international mathematics placement (Japan continues to place among the top 3), to quality mathematics content based on world standards. New problems are posed each week.

http://www.pacific.net/~mndel/

This web site features teachers helping teachers with ideas for mathematics instruction.

http://www.saxonpub.com

A mathematics test designed for students from 4th to 8th grade is available on this site. Saxon publishes a curriculum for home schoolers.

gardnerj@ucs.orst.edu

John Gardner, Physics Department, Oregon State University. Developed *Graphics: An Overview and Resource Guide* and currently developing DotsPlus, technology for producing tactile graphics.

Retail Companies and Organizations

Alphatek
1223 Wilshire Blvd.
Santa Monica, CA 90403
310-393-7780
clocks, watches, calculators

American Action Fund for Blind Children and Adults
18440 Oxnard St.
Tarzana, CA 91356
818-343-2022
calendars, Twin Vision books, handbook of braille contractions

American Printing House for the Blind
1839 Frankfort Ave.
P.O. Box 6085
Louisville, KY 40206-2405
800-223-1839 or 502-895-2405
educational products, braille writing equipment
Tactile Graphics Guidebook and Kit
Tangible Graphs Teacher's Guide and students' materials
Introduction to Map Study I and II

American Thermoform Corporation
2311 Travers Ave.
Commerce, CA 90040
213-723-9021
braille paper, labels, equipment

Ann Morris Enterprises, Inc.
890 Fams Ct.
East Meadow, NY 11554-5101
800-454-3175 or 516-292-9232
FAX: 516-292-2522
watches, clocks, timers, writing equipment, etc.

Beyond Sight, Inc.
26 East Arapahoe Dr.
Littleton, CO 80122
303-795-6425
FAX: 303-795-6425

Blazie Engineering
105 E. Jarrettsville Rd.
Forest Hill, MD 21050
410-893-9333
Braille 'N Speak; Type 'N Speak; Braille Lite; Graph-It

Carolyn's Products for Enhanced Living
P.O. Box 743
Brookfield, WI 53008-0743
800-648-2266
games, calculators, calendars, etc.

Childcraft
20 Kilmer Rd.
Edison, NJ 08818
201-572-6100 or 1-800-631-5657
games, manipulatives

Computers to Help People, Inc.
John J. Boyer
825 East Johnson Stree.
Madison, WI 53703
608-257-5917
e-mail: 76025.1265@compuserve.com.
Technical Braille Center; will produce highly technical material in braille or in a special file format

Cuisenaire Company of America, Inc. (Addison Wesley Longman Supplementary Division)
P.O. Box 5026
White Plains, NY 10602-5026
800-237-3142
FAX: 800-551-RODS
mathematics resource materials; excellent manipulatives for teaching attributes, fractions, geometry, etc.

Dale Seymour Publications
P.O. Box 10888
Palo Alto, CA 94303
Workbook series on Mental Math, Estimation and other topics in mathematics

Department of Chemistry
East Carolina University
Greenville, NC 27858-4353
Margaret M. Gemperline, MS
919-328-1648
FAX: 919-328-6210
e-mail: chmgempe@ecuvm.cis.ecu.edu
David Lunney, Ph.D., P.E.
919-758-6453
FAX: 919-758-0967
e-mail: LUNNEY@delphi.com

Don Johnston, Inc.: Products for Special Needs
1000 N. Rand Rd., Bldg. 115
Wauconda, IL 60084-0639
800-999-4660
FAX: 847-526-4177
MathLine manipulative (combination number line and abacus)

Easier Ways
1101 N. Calvert St., Suite 405
Baltimore, MD 21202
710-659-0232
FAX: 410-469-0233
plastic index cards, labels, and other labeling products

Exceptional Teaching Aids
20102 Woodbine Ave.
Castro Valley, CA 94546
800-549-6999
FAX: 510-582-5911
games, manipulatives, measuring devices, etc.
Workjobs activity books
Mangold Math #1 Readiness Kit

Howbrite Solutions, Inc.
12254 25th St. S.W.
Cokato, MN 55321
800-505-Mathematics
MathLine manipulative (combination number line and abacus)

Independent Living Aids, Inc.
27 E. Mall
Plainview, NY 11803-4404
800-537-2118 or 516-752-8080
calendars, watches, timers, etc.

Iowa State University
Media Resources Center
Ames, Iowa 50011
515-294-8022
Videos on tactile graphics
1. Preparing Tactile Adaptations for Mathematics and Science, Feb 1996
2. Using Adaptations for Mathematics and Science, July 1996

Library of Congress
National Library Services for the Blind and Physically Handicapped
1291 Taylor St. N.W.
Washington, D.C. 20542
800-424-8567 or 202-707-5100
taped books

Lighthouse Consumer Products
36-02 Northern Blvd.
Long Island City, NY 11101
800-829-0500
watches, clocks, calculators, etc.

Logan Electric Specialty Manufacturing Co.
1431 West Hubbard Street, Chicago, IL 60622
LS&S Group, Inc.
P.O. Box 673
Northbrook, IL 60065
800-468-4789 or 487-498-9777
FAX: 847-498-1482
watches, clocks, calculators, etc.

Maxi-Aids Aids and Appliances
42 Executive Blvd.
P.O. Box 3029
Farmingdale, NY 11735-0673
800-522-6294 or 516-752-0521
FAX: 516-752-0689
games, calendars, clocks, watches, protractor, slate, etc.

Mostly Mobility
7100 Route 183
Bethel, PA 19507
717-933-5681 (telephone and FAX)
e-mail: golds2@1xnetcom.com
"Abacus Attack" mathematics game for blind and visually impaired students

MPI Essential LifeSkills
P.O. Box 24155
1200 Keystone Ave.
Lansing, MI 48909-4155
800-444-1773 or 517-393-0440
FAX: 517-393-8884
games, manipulatives

National Braille Press
88 St. Stephen St.
Boston, MA 02115
617-266-6160
calendars

Pro-Ed
8700 Shoal Creek Blvd.
Austin, TX 78757-9965
"Orders Only" FAX: 800-397-7633
games, activities

Radio Shack
(local distributors)
calculators

Raised Dot Computing (David Holladay)
408 S. Baldwin St.
Madison, WI 53703
800-347-9594
FAX: 608-257-4143
MegaDots (braille translator/word processor; Baby Nemeth; beta testing advanced Nemeth translation program)

Repro-Tronics, Inc.
75 Carver Ave.
Westwood, NJ 07675
800-948-8453 or 201-722-1880
FAX: 201-722-1881
Bumpy Gazette: The Newspaper for Information on Tactile Imaging; source for the Tactile Image Enhancer, Thermo-Pen, and Flexi-Paper; TAG (Tactile Audio Graphics): AudioCAD (CAD scaled drawing system for blind people); AudioPIX (audio-tactile graphics reading system); AudioTRIP (real-time trip preview, virtual travel, O&M training system); & TraceME (tracing system for sighted people to create graphics that can be embossed in a graphics printer or with the Tactile Image Enhancer)

Royal National Institute for the Blind
Customer Services
P.O. Box 173
Peterborough PE20WS
England
44-733-370777
FAX: 44-733-371555
adapted games, appliance, other materials

Science Products for the Blind
Box 888
Southeastern, PA 19399
800-888-7400
talking calculators, etc.

Speak to Me!
17913 108th Ave. SE
Suite 155
Renton, WA 98055
800-248-9965
talking clocks, calculators, toys

APPENDIX E

REFERENCES

REFERENCES

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APPENDIX F

SURVEY RESPONDENTS
AND
FOCUS GROUPS

SURVEY RESPONDENTS

Mary Fran Anderson: Portland, Oregon
Norman Anderson: Baltimore, Maryland
Sharon L. Anderson: Medford, Oregon
Jill C. Brown: Ft. Worth, Texas
Katherine H. Brown: Raleigh, North Carolina
Jennifer Broxton: Skokie, Illinois
Karen Carl: O'Fallon, Missouri
Norman Coombs: Rochester, New York
Laura G. Cooper: St. Louis, Missouri
Chrissy Cowan: Austin, Texas
Michael Czerwinski: Egg Harbor, New Jersey
Annette Daniels: Muskogee, Oklahoma
Louise A. Esau: Mission, Texas
Robin Finley: Columbus, Ohio
Kathie Frankel-Mislinski: Tucson, Arizona
Nancy Frankl: Dover, Delaware
John Gardner: Corvallis, Oregon
Kathleen Geiger: Beaumont, Texas
Nancy Getten: Great Falls, Montana
Steve Jacobson: Minneapolis, Minnesota
Kenalea Johnson: Lubbock, Texas
Marilyn Johnson: Little Rock, Arkansas
Philip R. Johnson: Cedar Hill, Texas
Karen Kahler: Skokie, Illinois
Elaine Karns: Kansas City, Missouri
Carol Kaufman: Portland, Oregon
Kelly Kerr: Silverdale, Washingon
Marie Knowlton: Minneapolis, Minnesota
Linda Leo/Norris: Lewiston, Maine
Mia Lipner: Seattle, Washington
Marsha Lloyd: Williamsport, Pennsylvania
Janice Milusich: Hicksville, New York
Sarah Mitchell: St. Louis, Missouri
Barbara Maher: Palo Alto, California
Paulette Kamenista: Austin, Texas
Mary Kidwiler Moritz: Clear Lake, Iowa
C. Mason: Austin, Texas
Abraham Nemeth: Southfield, Pennsylvania
Frederick Otto: Louisville, Kentucky
Karen Ross: Newton, Massachusetts
Alan Roth: Vancouver, Washington
Betty Salz: Flushing, New York
Chaim Segal: Trotwood, Ohio
Margueite Sgrillo: Vallejo, California
Barbara Shaw: Austin, Texas

Margaret Spilker: Flagstaff, Arizona
Margaret Spittler: Colden, New York
Betty Squires: Little Rock, Arkansas
Mila Truan: Lebanon, Tennessee
Carolyn Vestal: Norman, Oklahoma
Gerald G. Weichbrodt: Livonia, Michigan
Cindy Wenrich: Pulaski, Virginia
Cheryl Williams: Hico, Texas
Stuart Wittenstein: Fremont, California
Nettie Wolf: Louisville, Kentucky
Gary L. Wright: Little Rock, Arkansas

ILLINOIS FOCUS GROUP

Debbie Augustine: Palatine, Illinois
Ann Dansdill: Wheaton, Illinois
Beverly Eisenhut: Downers Grove, Illinois
Diane Haisch: Montgomery, Illinois
Janet Huff: Bartlett, Illinois
Kathy Kinsey: Chicago, Illinois
Sue Schieve: Chicago, Illinois
Carol Shepardson: Bolingbrook, Illinois
Carolynn Werline: Sycamore, Illinois
Sheila Wexler: Skokie, Illinois

TEXAS FOCUS GROUP

Jeri Cleveland: Austin, Texas
Kathleen Ford: San Antonio, Texas
Carolyn Mason: Austin, Texas
Donna McNeer: Cambridge, Minnesota
Aimee Ormiston: Austin, Texas
Susan Osterhaus: Austin, Texas
Barbara Shaw: Austin, Texas
Renae Shepler: Austin, Texas
Glenda Torrence: Austin, Texas

VIRGINIA FOCUS GROUP

Sarah Campbell: Herndon, Virginia
Marsha Nicolai: Anandale, Virginia
Donna Pastore: Arlington, Virginia
Susan Ribyat: Alexandria, Virginia
Billy Ritter: Monassis, Virginia
Erica Spaulding: Arlington, Virginia
Sandra Sullivan: Falls Church, Virginia
Anna Swenson: Herndon, Virginia
Louise Watson: Arlington, Virginia

The Computerized Nemeth Code Tutor

Gaylen Kapperman
Jim Henry
Mario Cortesi
Toni Heinze
Jodi Sticken

August, 1997

Research and Development Institute
1732 Raintree, Sycamore, IL 60178

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Credit is due to all of the members of the team who developed this first of its kind tutorial. Those individuals are listed below with a brief description of their contributions.

Jim Henry's splendid programming skills were instrumental in the development of the underlying software. Without his untiring efforts, this project could not have been completed.

Mario Cortesi developed the content of the lessons. Intrepidly, he forged forward where no one had gone before. While shouldering this heavy responsibility, he also entertained all of his fellow team members with a very unique brand of humor.

Toni Heinze and Jodi Sticken committed many, many hours to the arduous task of editing the content. Their expertise in this work was invaluable.

The panel of experts composed of Stuart Wittenstein, Joe Wasserman, and Susan Osterhaus spent hundreds of hours reviewing our work. We profited greatly from their advice and counsel.

CaraLynn Pender reviewed the final version of the tutorial, giving us the benefit of her experience as a Nemeth Code transcriber. For that, we are appreciative.

We were extraordinarily fortunate to have Gloria Buntrock (in our estimation, the foremost Nemeth Code transcriber in the nation), as our chief advisor on the code. Her vast knowledge and experience were invaluable aids in the effort to develop this tutorial.

Finally, the data entry specialists, Ted Hines, Janet Huff, and Julie Hart spent many a long hour hunched over a computer, inputting Mr. Cortesi's masterpiece.

Ms. Hart deserves special recognition for her work beyond the call of duty, as it were.

It is our aim to provide an effective vehicle through which sighted persons who work with blind students can hone their skills in the Nemeth Code. As a consequence, eventually, more blind persons will be able to read and write braille mathematics, and thus, to be better prepared to participate in a technological society.

GK

INSTALLATION PROCEDURES

This program requires Microsoft Windows 3.1 or Windows 95. It consists of three diskettes. You must install them in the prescribed order.

Please read through these notes before beginning the installation.

To install the Computerized Nemeth Code Tutor, put the disk labeled #1 into your disk drive, and execute the program called setup.exe:

Select **Run . . .** from the program manager's File Menu (Windows 3.1) or from the Start Menu (Windows 95).

Type **a:\setup.exe** into the box labeled "Command Line" (Windows 3.1) or "Open" (Windows 95) and then press **Enter.** (Note: if the disk is in the b:drive, type b:\setup.exe).

Alternately, you can execute the setup program from the File Manager (Windows 3.1) or Explorer (Windows 95).

Follow the instructions and the installation program will install the program.

The default location is **c:\rdimath**, but you can choose an alternate location. See notes below if you choose to do so.

Notes

Required Disk Space: The Windows file system requires that each file must be allocated a specific amount of space. The requisite amount ranges from 512 characters to approximately 32,000 characters depending on the size of your hard disk. The Computerized Nemeth Code Tutor contains a very large number of small files, and thus the

space requirement is considerable. The total amount of space required will range from approximately 13 megabytes to 160 megabytes. Paradoxically, the smaller your hard disk, the less space the files will require. You can install the program to a relatively small removable drive (such as a Zip drive) to minimize the space required. Alternately, you can use a program such as Partition Magic to create a small (20 megabyte) partition for these files, and install the program in that partition.

The setup program will inform you that the system requires 4.7 megabytes. This is erroneous information. The following guidelines can be used to estimate the amount of space required.

- 30 megabyte partition—approximately 13 megabytes
- 340 megabyte hard drive—approximately 40 megabytes
- 2 gigabyte hard drive—approximately 160 megabytes

Installing to a directory other than c:\rdimath: If you choose to install to a drive or path other than c:\rdimath, you must make one manual change after installation:

1. Open Notepad
2. Load the file mathbr.ini, which was stored in your **\windows** subdirectory during installation
3. Locate the lines (at the beginning of the file)
   [Root]
   RootPath=c:\rdimath
4. Change **c:\rdimath** to the drive and directory you chose during installation
5. Save the file
6. Quit Notepad.

Math Symbol Display: On some video cards, certain of the math elements cannot be displayed correctly or, in some cases, they cannot be displayed at all. If you have difficulty with the display of math elements, you can change your video display driver to standard VGA, or you can change the color depth on your card. If these suggestions do not solve the problem, you must use a different computer with a different video card. Unfortunately, there are no alterations which can be made in the program to cause it to operate properly in these situations.

Uninstall: if you want to remove the Nemeth Tutor from your system, you can use the Uninstall program created during installation. It can be found in the Program Group (Windows 3.1) or Start Menu (Windows 95). However, the Uninstall program will **not** remove 4 small files. You can remove them manually if you so desire. They are:

- bwcc.dll (installed in windows\system; however, other programs may use this file)
- trbr.fot and trbr.ttf (the braille font: installed in \windows\system)
- mathbr.ini (installed in windows)

The uninstall program also leaves a set of empty subdirectories. These can be removed by selecting the rdimath subdirectory in File Manager (Win 3.1) or Explorer (Win 95) and pressing the delete key.

Known Problems:

1. Concurrent execution of other programs may rarely cause a problem to some parts of this program. If you encounter a "General Protection Fault," close all other running programs and restart the program (the only known instance of this is when running "Pegasus E-mail" and pressing the Tutor's opening screen "Credits" button).

2. Questions with 8 answer lines cause a message box to appear before the question. The message box instructs you to click on "OK." Do so and the question will be presented normally.

LESSON ORGANIZATION

The materials in this tutorial are organized into Lessons. Each Lesson has one or more Sections. Each Section has several Parts. Some Lessons may not include all possible Parts.

The Parts include:

- **Presentation:** New material is explained and illustrated here.
- **Print to Braille:** Inkprint exercises are presented here; you can respond by using the s, d, f and j, k, l keys on the computer keyboard to emulate a Perkins brailler keyboard. The program can "Judge" your answer and provide suggestions for correction.
- **Braille to Print:** In this Part, you are given braille to transcribe into print. You are instructed to write your answer on paper. Clicking "Show Answer" will then display the correct answer on the screen.
- **Proofreading:** In this Part, print content has been rendered incorrectly into braille. You are instructed to make the necessary corrections, and then check your answer using "Judge" or "Show Answer".
- **Quiz:** Exercises of the three types described above are presented within this portion of the Lesson.

Selecting Lessons, Sections, and Parts

- When you start the program, you will see the Opening Screen.
- Initially, you will see a pushbutton labeled "Lesson". Click it with the mouse.
- Next you will see a list of available Lessons. Choose one by clicking on it, and then click the "OK" button.
- You will see the Opening Screen again. Now a new pushbutton is visible, labeled "Section". Click it with the mouse.
- Next you will see a list of the Sections for the Lesson you just selected. Click on one, and then click the "OK" button.
- Now you will see a series of buttons showing you the Parts of this Section. Some may be "grayed out" (inactive), indicating that this Part is not supplied in this Section. Click on one of the active buttons, and you will see the content for this Part.
- When you are finished with this Part, click on the menu item "Opening Screen" to return to the Opening Screen. You then can choose a new Part, a new Section, or a new Lesson.

Typing a Braille Answer

- In any item that requires you to type a braille answer, you will type your response using the computer keyboard. The s, d, f and j, k, l keys correspond to the six braille dots. You can press them in any combination; the corresponding braille cell will be displayed on the screen after all the keys are released.

- You can move from one item to the next by pressing the TAB key.
- You can move the text insertion cursor by using the left and right arrow keys, or the HOME and END keys, or by clicking the mouse at any desired location to move the text insertion point to that location.
- You can use the BACKSPACE key to erase single characters to the left of the cursor, and the DELETE key to erase single characters to the right of the cursor.
- You can highlight multiple characters by pressing the SHIFT key while tapping the left or right arrow keys, or by moving the mouse while the right mouse button is depressed. The highlighted characters can then be deleted by pressing the DELETE key.

Feedback for Your Incorrect Answers

After you have entered a braille answer, you can press the menu item "Judge", to find out if your answer is accurate, and to get suggestions for correcting any errors.

If your response has more or fewer lines than the correct answer, a prompt will appear with instructions for correcting the problem.

Once a response has the correct number of lines, pressing "Judge" will cause the following symbols to be displayed below any errors:

- "s" (substitute) indicates that the Braille cell above the "s" is wrong. Replace it with the correct braille cell.

- "d" (delete) indicates that the cell above "d" should be deleted. It is probably an extra character.
- "i" (insert) indicates that there is a single braille cell missing. Add it.
- "^" (multiple insert) indicates that more than one braille cell is missing. Add them.

This feedback works best if your answer is only slightly wrong. If large portions of your response are in error, a confusing series of symbols (e.g. dsdd^) may appear. In this case, do not attempt to follow the suggestions literally. Review the response carefully to resolve significant errors. Your entire response may have to be rewritten.

REFERENCES

The following reference works were used in the preparation of this tutorial. Citations are indicated by abbreviations in parentheses, followed by section numbers [e.g., (AITBM, §30)]. The learner can refer to these sources for additional explanations and examples. In addition, the abbreviation (BANA) refers to the Braille Authority of North America which has provided valuable updates and revisions of braille codes.

(BANA) *Braille code for columned materials and tables* (1995). Louisville, KY: American Printing House for the Blind.

(CBTFAT) *Code of braille textbook formats and techniques* (1977). Louisville, KY: American Printing House for the Blind.

(LLTBNC) Craig, R. H. (1987). *Learning the Nemeth braille code: A manual for teachers and students.* Louisville, KY: American Printing House for the Blind.

(AITBM) Roberts, H., Krebs, B. M., & Taffet, B. (1978). *An introduction to braille mathematics*. Washington, DC: Library of Congress.

(NBC) *The Nemeth braille code for mathematics and science notation 1972 revision* (1973). Louisville, KY: American Printing House for the Blind.

Errata

1. In Lesson 6, Section 1, Print to Braille, Item 6, the separator line will not appear in the "Show Answer" portion. You will not be able to braille it because there is insufficient space available.
2. In Lesson 9, Section 2, EXMAPLES should be spelled EXAMPLES.
3. In Lesson 18, Section 3, the review section, BRAILLy should be spelled BRAILLE.
4. In each Lesson, when working on the Part entitled "Braille to Print", clicking on "Show Answer" will display the correct answer which you can compare to your answer written on paper. The "Judge" option will not work in this mode.